HOMEWORK 1

1) In Chapter 0 under the heading "Using Symbols" state numbers 3 and 8 in your own words.

2) In Chapter 0 under the heading "Common Words and Phrases" state numbers 3 and 6 in your own words.

3) Write each of the following sets by listing the elements with braces.
   a) $A = \{x \in \mathbb{R} : x^3 = 27\}$
   b) $B = \{n \in \mathbb{Z} : n^2 < 8\}$
   c) $C = \{x \in \mathbb{R} : x^4 + 2 = 0\}$
   d) $D = \{n \in \mathbb{N} : |n| \leq 2\}$

4) Write each of the following sets in the form $\{x \in S : p(x)\}$ where $p(x)$ is a property on x and $S = \{-4, -2, -1, 0, 1, 2\}$.
   a) $A = \{-1, 0, 1\}$
   b) $B = \{-4, -2, -1\}$
   c) $C = \{-1, 1\}$
   d) $D = \{-4, -2, 2\}$

5) Give examples of sets A, B, and C such that the following hold. In each example it must be clear to the grader that you understand the properties required. I.E.: If I cannot distinguish whether you understand the notation or not, your answer is not correct.
   a) $A \in B$ and $A \subset C$
   b) $A \subseteq B$, $A \in C$ and $A \cap C \neq \phi$
   c) $A \subseteq C$, $B \subset C$, and $A \cap B = \phi$
   d) $|A \cap B| = |A - B| = |B - A| = |\overline{A \cup B}| = 2$ and $C = U$ the universal set.

6) Let $A = \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$. (For $d - g$ determine the set indicated.)
   a) Determine $|A|$.
   b) List the elements of A.
   c) List all possible subsets of A.
   d) $\phi \cap A$
   e) $\{\{\phi\}\} \cap A$
   f) $\{\phi\} \cup A$
   g) $\{\phi, \{\{\phi\}\}\} \cup A$

7) Let $U = \{a, b, c\}$ be the universal set, and let $A = \{a, b\}$, $B = \{b, c\}$, and $C = \{a, c\}$. Determine the following:
   a) $(A \cup B) - (B \cap B)$
   b) $\overline{A}$
   c) $\overline{(B \cup C)}$
   d) $A \times B$

8) Which of the following are statements? State whether the statements are True or False.
   a) 6 is even.
   b) $7 - 1 = 8$
   c) $2x + 7 = 13$
   d) multiply 2 and 5

9) Which of the following are true? Explain each of your answers.
   a) $1 \subseteq \{1\}$
   b) $\phi \in \{2\}$
   c) $\phi \in \phi$
   d) $\phi \subset \{\phi\}$

10) Negate each of the following. You should remove 'not' from your sentences.
   a) π is not irrational.
   b) Zero is a positive number.
   c) Three is odd.
   d) 21 is composite.

11) Let P : 2 is odd and Q : 5 is prime. Determine whether the following are true or false. Justify each part.

   a) $P \vee Q$
   b) $P \wedge Q$
   c) $(\sim P) \wedge Q$
   d) $P \Rightarrow Q$
   e) $Q \Rightarrow P$
   f) $(\sim P) \Rightarrow Q$
   g) $P \Leftrightarrow Q$
   h) $(\sim P) \vee (\sim Q)$

12) Let $P : \sqrt{2}$ is rational and $Q : \frac{2}{6}$ is rational. Determine whether the following are true or false. Justify each part.

   a) $P \vee Q$
   b) $P \wedge Q$
   c) $P \Rightarrow Q$
   d) $Q \Rightarrow P$
   e) $(\sim P) \Rightarrow (\sim Q)$
   f) $(\sim Q) \Rightarrow (\sim P)$
   g) $P \Leftrightarrow Q$
   h) $(\sim P) \Leftrightarrow (\sim Q)$

13) In each of the following two open sentences $P(x)$ and $Q(x)$ are defined over a given domain S. Determine all $x \in S$ for which $P(x) \Rightarrow Q(x)$ is true.

   a) $P(x) : 3x + 1$ is prime; $Q(x) : 2x + 1$ is prime; $S = \{1, 2, 3, 4\}$
   b) $P(x) : x^2 = 4$; $Q(x) : |x| = 2$; $S = \{-3, -2, 0, 1\}$
   c) $P(x) : x^2 \geq 9$; $Q(x) : x \geq 3$; $S = \mathbb{R}$
   d) $P(x) : x^2 \geq 9$; $Q(x) : x \geq 3$; $S = \mathbb{N}$

14) In each of the following two open sentences $P(x, y)$ and $Q(x, y)$ are defined over a given domain S. Determine all $(x, y) \in S$ for which $P(x, y) \Rightarrow Q(x, y)$ is true.

   a) $P(x, y) : x^2 - y^2 = 0$; $Q(x, y) : x = y$; $S = \{(1, -1), (3, 2), (9, 9)\}$
   b) $P(x, y) : |x| = |y|$; $Q(x, y) : x = y$; $S = \{(-3, 3), (1, 1), (-1, 2)\}$

15) Repeat homework problem 13 with $P(x) \Leftrightarrow Q(x)$ instead of $P(x) \Rightarrow Q(x)$.

16) Repeat homework problem 14 with $P(x, y) \Leftrightarrow Q(x, y)$ instead of $P(x, y) \Rightarrow Q(x, y)$.

17) Give truth tables for the following: Label each as a Tautology, Contradiction, or Neither.
   a) $P \wedge (Q \Rightarrow (\sim P))$
   b) $(P \wedge Q) \Leftrightarrow P$ (what is this logically equivalent to?)

18) Verify the pairs of statements are logically equivalent.
   a) $P \Rightarrow (Q \vee R)$ and $(\sim Q) \Rightarrow ((\sim P) \vee R)$
   b) $(P \wedge Q) \Rightarrow R)$ and $(P \wedge (\sim R)) \Rightarrow (\sim Q)$

19) Negate the following: (whenever possible remove the 'not')
   a) Either x is even or y is even.
   b) Both a and b are zero.
   c) If $x < 0$, then $x^2 \leq 0$.
   For d and e, let A be subset of a universal set U.
   d) For every set A, $A \cap \overline{A} = \phi$.
   e) There exists a set A such that $\overline{A} \subseteq A$.
   f) There exists a rational number q such that $q^2 + q$ is rational.
   g) For all rational numbers q, $q^3 > 3$.

20) State each of the following in symbols and negate each statement (or open sentence).
   a) There exist integers a and b such that $ab < 0$ and $a + b > 0$.
   b) For all real numbers x and y, if $x + y < 0$, then $xy \leq 0$.

21) Determine if each of the following is True or False. Justify you answer.
   a) $\forall x \in \mathbb{Z}, x^2 > 0$
   b) $\exists n \in \mathbb{N}, n < 1$
   c) $\exists q \in \mathbb{Q}, \frac{1}{q^2} = \frac{1}{3}$
   d) $\forall m, n \in \mathbb{Z}, n + m > 0$
   e) $\forall m, n \in \mathbb{N}, n + m > 0$

Not collected Book problems: 1.1, 1.5, 1.7, 1.11, 1.19, 1.41, 2.1, 2.3, 2.5, 2.8, 2.11, 2.17, 2.19, 2.23, 2.33, 2.37, 2.39, 2.45, 2.49