HOMEWORK 11

1) Determine whether the following are true or false. Justify your answers.
   a) If the sequence $\{a_n^2\}$ converges, then $\{a_n\}$ converges.
   b) If the sequence $\{a_n + b_n\}$ converges, then $\{a_n\}$ and $\{b_n\}$ converge.
   c) If the sequence $\{|a_n|\}$ converges, then $\{a_n\}$ converges.

2) Prove that if the sequences $\{a_n\}$ and $\{b_n + a_n\}$ converge, then the sequence $\{b_n\}$ converges also.

3) Prove that $\{c_n\} \to c$ if and only if $\{c_n - c\} \to 0$.

4) Fitzpatrick section 2.1 number 11.

5) Fitzpatrick section 2.1 number 17.

6) Suppose the sequence $\{a_n\}$ converges to a and α is non-zero real number. Prove the sequence $\{\alpha a_n\}$ converges to αa by:
   a) using (applying) the Comparison Lemma.
   b) by definition of convergence. (Please note the proof should be similar to the comparison lemma's proof.)

7) Prove the product property for sequences. You may not follow the proof from Fitzpatrick. You should use the technique of adding/subtracting the same thing. So $a - b = a - c + c - b$. You will also need the triangle inequality and 'facts' about convergent sequences.

8) Chartrand number 12.41.

9) Using the definition of convergence (or its negation) prove the sequence $\{\frac{2n-5}{n+3}\}$ does NOT converge to 1.