

HOMEWORK 12

- 1) Determine whether the following are true or false. Justify your answers.
 - a) Every bounded sequence converges.
 - b) A convergent sequence of positive numbers has a positive limit.
 - c) The limit of a convergent sequence in (a, b) also belongs to (a, b) .
- 2) Suppose the sequence $\{a_n\}$ converges to $a < 1$. Prove there exists an index N such that for all $n \geq N$, $a_n < 1$.
- 3) Let $\{a_n\}$ and $\{b_n\}$ be convergent sequences where there exists an index N such that $a_n \geq b_n$ for all $n \geq N$. Prove $\lim a_n \geq \lim b_n$. [Note: If you change all of the \geq to $>$ can you still do this?]
- 4) Prove the set $[2, 5) \cup (5, 8]$ is not closed.
- 5) Prove the set $[0, \pi]$ is closed.
- 6) Let $a, b \in \mathbb{R}$ where $a \leq b$. Prove $[a, b]$ is closed.
- 7) Let $b, L \in \mathbb{R}$. Prove if $b \geq L - \epsilon$ for all positive ϵ , then $b \geq L$.
- 8) Prove every real number is the limit of a sequence of irrationals.
- 9) Let S be a (non-empty) bounded set of real numbers. Prove there are sequences (most likely distinct!) in S that converge to $\sup S$ and $\inf S$.