HOMEWORK 13

1) Fitzpatrick Section 2.3 number 1.

2) Assume $\{a_n\}$ is monotonically increasing and define $s_n = a_1 + a_2 + \cdots a_n$ for all $n \in \mathbb{N}$. Prove the sequence $\{\frac{s_n}{n}\}$ is also increasing. [Note this can also be done for a decreasing sequence.]

3) Use the Monotone Convergence Theorem to prove each of the following sequences converge.
   a) $y_n = \frac{3n-1}{n-2}$ for all $n \in \mathbb{N}$.
   b) $x_n = \frac{1}{2n+1} + \frac{1}{2n+2} + \cdots + \frac{1}{3n}$ for all $n \in \mathbb{N}$ (Hint writing $3n$ as $2n+n$ might help.)
   c) $\{a_n\}$ by $a_n = \frac{\sqrt{n}}{n+2}$ for all $n \in \mathbb{N}$

4) Fitzpatrick section 2.3 number 3.

5) Fitzpatrick section 2.3 number 8.

6) Fitzpatrick Section 2.3 number 9.

7) Let $0 < r < 1$.

   a) Prove $\{r^n\}$ converges using the Monotone Convergence Theorem.
   b) What does $\{r^n\}$ converge to? Prove your answer is correct using the definition of convergence.

8) Fitzpatrick Section 2.4 number 3.

9) Fitzpatrick Section 2.4 number 9.

10) Let $\{a_n\}$ converge to a. Prove all subsequences of $\{a_n\}$ also converge to a.

11) Let $\{a_n\}$ be a sequence. Let $\{b_{n_k}\}$ and $\{c_{n_k}\}$ be subsequences of $\{a_n\}$. Prove directly (you may not quote the previous question) that if $\{b_{n_k}\}$ converges to b and $\{c_{n_k}\}$ converges to c with $b \neq c$, then $\{a_n\}$ does not converge to c. (Note, b and c are interchangeable and this proof extends to $\{a_n\}$ does not converge to any real number, but the algebra is more complicated.)