HOMEWORK 2

1) Let $x \in \mathbb{R}$. If $x^3 > 0$, then $x^2 + 1 > 0$.

2) Let $y \in \mathbb{R}$. If $y^2 < 0$, then $y + 17 > 0$.

3) Prove directly:
   a) Let $x \in \mathbb{Z}$. If x is even, then $3x - 2$ is even.
   b) Let $x \in \mathbb{Z}$. If x is odd, then $3x - 2$ is odd.
   c) Let $n \in \mathbb{Z}$. If $n^2 - 1 < 0$, then $n - 2$ is even.

4) Prove by contrapositive:
   a) Let $x \in \mathbb{Z}$. If $3x - 1$ is even, then x is odd.
   b) Let $x \in \mathbb{Z}$. If $3x - 1$ is odd, then x is even.
   c) Let $n \in \mathbb{Z}$. If $n^2 - 1 < 0$, then $n - 2$ is even.

5) Prove that $x + y$ is even if and only if x and y are of the same parity.

6) Prove that ab and $a + b$ are the same parity if and only if a is even and b is even.

7) Prove that if $n \in \mathbb{Z}$, then $n^3 + n$ is even.

8) Let $m \in \mathbb{Z}$. Prove that $5|m$ if and only if $5|m^2$.

9) Let $x, y \in \mathbb{Z}$.

   a) Prove that if $3 \nmid x$, then $x^2 = 3m + 1$ for some $m \in \mathbb{Z}$.

   b) Using part a, Prove that if $3 \nmid x$ and $3 \nmid y$, then $3|(x^2 - y^2)$.

10) Let $x, y \in \mathbb{Z}$. Prove that $5|xy$ if and only if $5|x$ or $5|y$.

Not collected Book problems: $3.1, 3.5, 3.7, 3.11, 3.12, 3.20, 3.27, 3.28, 3.29, 3.30, 3.33, 3.39, 4.3, 4.5, 4.7$