HOMEWORK 5

1) Disprove that if A and B are sets such that $A \cap B = \phi$, then $A = \phi$ or $B = \phi$.

2) Prove that if A and B are sets such that $A \cup B \neq \phi$, then $A \neq \phi$ or $B \neq \phi$.

3) Let A, B, and C be sets.
   a) Define A, B, and C such that $A \cap B = A \cap C$, but $B \neq C$.
   b) Define A, B, and C such that $A \cup B = A \cup C$, but $B \neq C$.
   c) Prove that if $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then $B = C$.

4) Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$. (You are to prove this part of Thm 4.21.)

5) Let A and B be sets. Prove that $A \subseteq B$ if and only if $A \cup B = B$.

6) Let A and B be sets. Prove that if $B - A \neq \phi$, then $B \nsubseteq A$.

For questions $7 - 9$ you need to write the statement in quantifiers, negate the statement, and then Prove or Disprove the statement as denoted at the end of the question.

7) There exist integers m and n such that for every natural number k, $m < \frac{1}{k} < n$. [Prove]

8) For all real numbers x and y, there exists an integer n such that, $x < n < y$. [Disprove]

9) There exists a positive integer n such that for every integer m, $nm > 0$. [Disprove]

Not collected Book problems: $4.27, 4.33, 4.34, 4.37, 4.61, 4.64, 7.3, 7.6, 7.7, 7.11$