HOMEWORK 6

For problems 1 through 12 Prove or Disprove: (You must clearly state which you are doing and how!)

1) For every two rational numbers a and b with $a < b$, there exists a rational number c such that $a < c < b$.

2) If $x, y \in \mathbb{R}$ and $x^3 < y^3$, then $x < y$.

3) For every set A there exists a set B such that $A \cap B = \phi$.

4) For every two sets A and B, $(A \cup B) - B = A$.

5) Let A be a set. If, for all sets B, $A - B = \phi$, then $A = \phi$.

6) Let A be a set, then for all sets B, $A \cap B = \phi$ if and only if $A = \phi$.

7) Let $f(x) = x^5 + 2x^3 + 1$ be a polynomial defined on $\mathbb{R}$. Then $f(x)$ has a zero in $[-1, 1]$.

8) Let $f(x) = x^2 + 4x^4 + 1$ be a polynomial defined on $\mathbb{R}$. Then $f(x)$ has a zero in $[-1, 1]$.

9) For every positive integer n, $n^2 - n + 11$ is prime.

10) There exist two distinct positive integers whose sum exceeds their product.

11) There exists an odd integer, the sum of whose digits is odd and the product of whose digits is even.

12) There exists an even integer, the sum of whose digits is even and the product of whose digits is odd.

13) State if the following relations on $A = \{a, b, c\}$ are reflexive, symmetric, and/or transitive (or not).
    a) $R_1 = \{(a, a), (b, b), (c, c), (a, c), (c, b)\}$.
    b) $R_2 = \{(c, c), (a, b), (b, c), (a, c)\}$.
    c) $R_3 = \{(a, a), (b, c), (c, a), (c, b), (a, c)\}$.

14) Create non-empty relations S and T on the set $A = \{r, s, t\}$ that satisfy the following:
    a) S is Not reflexive, is symmetric and is transitive.
    b) T is Reflexive, is Not symmetric and is transitive.

Not collected Book problems: $7.7, 7.11, 7.13, 7.25, 7.26, 7.27, 7.32, 7.37, 7.41, 7.52, 7.54, 7.60$