

HOMEWORK 8

For questions 1 through 4: Let A, B, C be non-empty sets and $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions. Remember a picture is NOT a proof, but it can be an example or counterexample.

- 1)
 - a) Prove: If $g \circ f$ is injective, then f is injective.
 - b) Disprove: If $g \circ f$ is injective, then g is injective.
 - c) Prove or disprove: If $g \circ f$ is surjective, then f is surjective.
 - d) Prove or disprove: If $g \circ f$ is surjective, then g is surjective.

- 2) Prove or disprove:
 - a) If f is injective, then $g \circ f$ is injective.
 - b) If f is surjective, then $g \circ f$ is surjective.
 - c) If g is injective, then $g \circ f$ is injective.
 - d) If g is surjective, then $g \circ f$ is surjective.

- 3) Prove or disprove:
 - a) There exists functions f and g such that f is not injective and $g \circ f$ is injective.
 - b) There exists functions f and g such that g is not injective and $g \circ f$ is injective.
 - c) There exists functions f and g such that f is not surjective and $g \circ f$ is surjective.
 - d) There exists functions f and g such that g is not surjective and $g \circ f$ is surjective.

- 4) ***More difficult*** Let $g \circ f$ be bijective. Prove:
 - a) If f is surjective, then g is injective.
 - b) If g is injective, then f is surjective.
 - c) How can we combine a and b into one statement?

- 5) Textbook question 9.37

- 6) (PHP) Let S be a subset of size $n + 1$ of the set $\{1, 2, 3, \dots, 2n\}$. Prove S contains a pair of relatively prime numbers. Give an example of a set of size n where this is false.

- 7) Textbook question 10.6

- 8) Repeat textbook question 10.6 with the function onto instead of one-one.

- 9) Textbook question 10.19

- 10) Textbook question 10.24

Not collected Book Questions: 9.15, 9.17, 9.19, 9.21, 9.24, 9.29, 9.30, 9.43, 9.48($a-f$), 10.1, 10.3, 10.30, 10.33