HOMEWORK 9

1) For each of the following sets find the infimum and supremum. Where necessary you should state one or both do not exist (DNE). You do not need to prove your answers are correct.
   a) $\{x \in \mathbb{R} | x^2 > 3\}$
   b) $\{\frac{1}{n} | n \in \mathbb{Z}, n \neq 0\}$
   c) $\{x \in \mathbb{Z} | x + 4 < \pi\}$
   d) Let a be a fixed real number. $\{q \in \mathbb{Q} | q > a^2\}$

2) Determine if the following are true or false. Justify your answers.
   a) The set of negative rational numbers has an infimum.
   b) Every nonempty set of real numbers that is bounded above has a greatest element.
   c) If S is a nonempty set of negative real numbers, then $\sup S \leq 0$.
   d) $\mathbb{Z}$ is dense in $\mathbb{R}$.
   e) $\mathbb{Q} - \mathbb{Z}$ is dense in $\mathbb{R}$.

3) Let S be a nonempty set of real numbers that is bounded. Prove the following:
   a) If B is a nonempty subset of S, then $\sup S \geq \sup B$ and $\inf S \leq \inf B$.
   b) $\inf S \leq \sup S$.
   c) If $\inf S \geq \sup S$, then S has only one element.

4) Fitzpatrick Section 1.1 number 11.

5) Let S be a nonempty set of real numbers that is bounded below. Prove that S has a minimum if and only if the value $\inf S$ belongs to S.

6) Let a be a real number that has the property that $a \leq \frac{1}{n}$ for all natural numbers n. Prove $a \leq 0$.

7) Let b be a real number and define $S = \{x \in \mathbb{I} | x < b\}$. Prove $\sup S = b$.

8) Let b be a real number and define $S = \{x \in \mathbb{Q} | x > b\}$. Prove $\inf S = b$.

9) a) Fitzpatrick Section 1.3 number 14.
   b) Fitzpatrick Section 1.3 number 15.
   c) Fitzpatrick Section 1.3 number 17a.
   d) Fitzpatrick Section 1.3 number 17b.