Math 401 - Section 0301 - Spring 2016 **Instructor: Ricardo H. Nochetto**

MATLAB Project # 2 – Due Date: 04/26/2016

You must work each problem in a separate M-file and hand in a printout for each problem.

Problem 1 (30 pts) *Data Fitting*. The following data are the US census for the population (in millions) in the USA between 1900 and 1990:

1900	1910	1920	1930	1940
75.995	91.972	105.711	123.203	131.669

1950	1960	1970	1980	1990
150.697	179.323	203.212	226.505	249.633

Proceed as follows to find the least squares fit $p(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3$ to the given table using MATLAB.

(a) Determine the rectangular matrix A and right-hand side $\mathbf{b}$ of the least squares problem.

(b) Form and solve the Normal Equations. Use the commands `A'` to transpose A and \ to solve the linear system by Gaussian elimination. Plot the table and $p(t)$ for $1900 \leq t \leq 1990$ together.

(c) Show that $A^T A$ is symmetric and positive definite, that is $\mathbf{x}^T A^T A \mathbf{x} > 0$ for all $\mathbf{x} \in \mathbb{R}^n$, provided A is full rank ($A \in \mathbb{R}^{m \times n}$ with $n \leq m$). Use the command `rank(A)` to find the rank of A of (a). Compute the Cholesky decomposition of $A^T A$, using the command `R = chol(A'*A)` (see Olver-Shakiban p.168), and then solve by backward and forward substitution (use the command \).

(d) Find the QR factorization of A with the commands `[Q,R]=qr(A)` and solve the least squares problem. Plot the table and $p(t)$ for $1900 \leq t \leq 1990$ together.

Problem 2 (30 pts) *Data Fitting and Orthogonal Polynomials*. This problem repeats Pb#1 but replacing the canonical basis $\{1, t, t^2, t^3\}$ of $\mathbb{P}^3$ by orthogonal polynomials.

(a) Obtain orthogonal polynomials $\{p_1(t)\}_{i=0}^3$ with respect to the scalar product $\langle p, q \rangle = \int_a^b p(t)q(t)dt$ where $a = 1900$ and $b = 1990$. Determine by hand the first four Legendre polynomials on the interval $[-1, 1]$ by means of the Gram-Schmidt orthogonalization procedure, and then transform them to the interval $[1900, 1990]$ by the simple change of variables $x = (t - 1945)/45$. Explain why the resulting polynomials are still orthogonal.

(b) Repeat Pb#1 (b). Use the command `cond(A'*A)` to find the condition number of $A^T A$ and compare with that in Pb#1(b) (recall ML#1-Pb3(c) for the meaning of `cond`). Draw conclusions.

(c) Repeat Pb#1 (d).

Problem 3 (40 pts) *Fast Fourier Transform and Denoising*. This problem shows how a noisy signal can be transformed to the frequency domain via the Fast Fourier Transform (FFT), clean via thresholding of the smallest coefficients, and transform back to obtain a signal with less noise.

(a) Given the equally spaced 2^8 sampling points `x = [1:256]*2*pi/256` in the interval $[0, 2\pi]$, consider the function values `y = sin(5*x)` and the *noisy* function values `z = sin(5*x) + 0.1*randn(256,1)'`. The command `randn(256,1)` generates a normal distribution of 256 random numbers with zero mean and variance one. Use `plot(x,y,x,z)` to plot both functions.

(b) Compute `f = fft(z)`. Write a MATLAB function `t = thresh(f,a)` which computes the modulus of each component of `f` using the commands `conj` and `sqrt`, and then zeros all entries with values `< a`.

(c) Compute `s = ifft(t)`, the inverse FFT of `t`, for `a = 2, 3, 4`. Plot the three cases using `plot(x,s)` and draw conclusions.