MATH 462 Section 0201 Fall 2018

Partial Differential Equations

HOMEWORK # 11 (due Tu Dec 4)

1 (25 pts). Problem 3 in §6.1 of Strauss.
(a) Find the ODE satisfied by radial solutions of $-\Delta u = -u_{xx} - u_{yy} = k^2 u$ with $k > 0$ (note the different sign of Δ with respect to the book)
(b) Relate this ODE with the Bessel ODE of §10.2 of Strauss (p.266 of 2nd edition and p.252 of 1st edition). To this end you need to make a change of variables $\rho = kr$. This will convert (10) in p.266 with $n = 0$ into your ODE.
(c) One solution to (10) in p.266 for $n = 0$, bounded at the origin, is the Bessel function of the first kind $J_0(\rho)$. Another solution, unbounded at the origin, is the Bessel function of the second kind $K_0(\rho)$. They are linearly independent and span the set of solutions of (10) in p.266 for $n = 0$. Plot these functions using the built-in MATLAB functions `besselj(0,x)` and `besselk(0,x)` in the interval $0 \leq x \leq 20$.

2 (25 pts). Problem 8 in §6.1 of Strauss.
(a) Assume the solution is radial and find the governing ODE and boundary conditions.
(b) Integrate this ODE twice. First integrate from r to b, and next from a to r. Use the boundary conditions in both cases.
(c) Once you have an explicit expression for the solution in terms of powers of r consider the limit $a \to 0$. Interpret the result.

3 (25 pts). Problem 10 of §6.1 of Strauss.
(a) Let $w = u_1 - u_2$ be the difference of two solutions. Write the Divergence Theorem for the vector field $w\nabla w$. Compute explicitly div $(w\nabla w)$.
(b) Argue with the resulting term $|\nabla w|^2$.

4 (25 pts). Problem 3 of §6.2 of Strauss.