MATH 462 Section 0101 Spring 2008

Partial Differential Equations for Scientists and Engineers

HOMEWORK # 13 (this is for practice for the final exam only)

1. Problem 3 of §6.2 of Strauss.

2. Problem 1 of §6.3 of Strauss.

3. Prove that second order centered differences are second order accurate, namely

$$\frac{u(x_{j-1}) - 2u(x_j) + u(x_{j+1})}{\Delta x^2} - u''(x_j) = C\Delta x^2 u^{(4)}(\xi),$$

and find explicitly the numerical constant C. Hint: use Taylor expansion around $x = x_j$.

4. Problem 3 in §8.2 of Strauss.

5. Problem 7 in §8.2 of Strauss.

6. Problem 11 of §8.2 of Strauss.

7. Problem 3 in §8.5 of Strauss: Consider the approximation of the 2-point boundary value problem

$$-au'' + bu = f(x) \quad 0 < x < 1, \qquad u(0) = u(1) = 0,$$

by continuous piecewise linear *finite elements* over a uniform partition of $[0, 1]$ with mesh-size h; $a > 0, b \geq 0$ are constants. Let $u_h(x) = \sum_{i=1}^{N} U_i \phi_i(x)$ be the finite element solution.
(a) Write the system of equations $A\mathbf{U} = \mathbf{F}$ for the vector $\mathbf{U} = \{U_i\}_{i=1}^{N}$ of nodal values of u_h, that is find the matrix A and right-hand side $\mathbf{F}$.
(b) Compare the resulting system with that associated with centered finite differences.