1 (25 pts). Show an energy identity for the heat equation with convection and Dirichlet boundary condition along the lines of page 44 of Strauss:

\[ u_t - ku_{xx} + Vu_x = 0 \quad 0 < x < 1, \quad t > 0 \]
\[ u(0, t) = u(1, t) = 0 \quad t > 0 \]
\[ u(x, 0) = \phi(x) \quad 0 < x < 1. \]

2 (25 pts). Let \( Q = (0, L) \times (0, T) \) be a parabolic cylinder. Let \( u = u(x, t) \) be a continuous function in the closure of \( \overline{Q} = [0, L] \times [0, T] \) of \( Q \), and have two continuous derivatives in space and one derivative in time in \( Q \). Let \( f = f(x, t) \) be continuous \( \overline{Q} \) and \( g \) be continuous in the parabolic boundary \( \partial_p Q \) of \( Q \). Let \( u \) be the solution of

\[ \partial_t u - k\partial_{xx} u = f \quad \text{in} \ Q \]
\[ u = g \quad \text{on} \ \partial_p Q. \]

Show that

\[ f \geq 0, \quad g \geq 0 \quad \Rightarrow \quad u \geq 0. \]

Interpret this result in physical terms, for instance assuming that \( u \) stands for temperature.

3 (25 pts). Problem 7 in §2.3 of Strauss.

4 (25 pts). Problem 8 in §2.3 of Strauss.