

PARTIAL DIFFERENTIAL EQUATIONS**HOMEWORK # 4** (due Th Sep 26)

1 (25 pts). Let $u(x, t)$ be the solution to the wave equation $u_{tt} - u_{xx} = 0$ for $x > 0$ with Robin boundary condition $u_x(0, t) = u(0, t)$ for $t > 0$. If $u(x, t)$ vanishes outside a bounded interval, then show that the following *energy* is conserved:

$$E(t) = \frac{1}{2}u^2(0, t) + \frac{1}{2} \int_0^\infty \left(u_t^2(x, t) + u_x^2(x, t) \right) dx.$$

2 (25 pts). Problem 5 in §2.3 of Strauss.

3 (25 pts). Problem 7 in §2.3 of Strauss.

4 (25 pts). Problem 8 in §2.3 of Strauss.