

**PARTIAL DIFFERENTIAL EQUATIONS****HOMEWORK # 4** (due Tue Sep 27)

1 (25 pts). Let  $u(x, t)$  be the solution to the wave equation  $u_{tt} - u_{xx} = 0$  for  $x > 0$  with Robin boundary condition  $u_x(0, t) = u(0, t)$  for  $t > 0$ . If  $u(x, t)$  vanishes outside a bounded interval, then show that the following *energy* is conserved:

$$E(t) = \frac{1}{2}u^2(0, t) + \frac{1}{2} \int_0^\infty \left( u_t^2(x, t) + u_x^2(x, t) \right) dx.$$

2 (25 pts). Problem 5 in §2.3 of Strauss.

3 (25 pts). Problem 7 in §2.3 of Strauss.

4 (25 pts). Problem 8 in §2.3 of Strauss.