This course builds on AMSC 466 (Introduction to Numerical Analysis I), which is a prerequisite. We thus assume familiarity with the following topics:

- Direct methods for linear systems: LU decomposition, norms, condition number
- Nonlinear equations: Newton’s methods for scalar and systems
- Polynomial interpolation: divided differences, Lagrange interpolation, error formula
- Quadrature: trapezoid and Simpson rules, composite rules.

The following books contain this material:


The goal of this course is to cover more advanced techniques and theory on four important topics described below. Moreover, this course can be used to fulfill the AMSC qualifying exam requirements: the sequences AMSC 666 - AMSC 714 (Numerical Methods for Stationary PDEs) or AMSC 666 - AMSC 715 (Numerical Methods for Evolution PDEs) are allowed. Some topics overlap with AMSC 660 and AMSC 661. The difference is that the emphasis in AMSC 666 is on numerical analysis whereas the emphasis in AMSC 660-1 is on scientific computation. These course are thus complementary.

### Topics

- **Approximation Theory** (3 weeks)
  - Vector, matrix, and function norms
  - Jackson theorems
  - Least squares, QR, SVD
  - Orthogonal polynomials
  - Gaussian quadrature

- **Numerical Solution of Ordinary Differential Equations** (4 weeks)
  - Consistency, stability, and convergence analysis
  - One-step methods, Runge-Kutta methods
  - Multistep methods
  - Methods for stiff ODEs
  - Symplectic integrators

- **Iterative Methods for Linear Algebraic Systems** (3 weeks)
• Motivation: boundary-value problems for elliptic PDEs
• Classic iterative methods
• Steepest descent and conjugate gradient methods, preconditioning

• **Optimization** (4 weeks)
  • Steepest descent, Newton, and Quasi-Newton methods
  • Line search and trust region methods
  • Rates of convergence
  • Nonlinear conjugate gradient method

**Literature**


**Grading Policy**

The final grade will be based on **homeworks (45%)**, one **mid-term exam (25%)**, and a **final exam (30%)**. Computer exercises will use MATLAB. Homeworks will be assigned weekly and will be due before class starts. There will be penalties for late homework.

**Mid-Term Exam:** (≈ Thursday October 24).
**Final Exam:** Monday, December 16, 4-6pm.

**Homeworks**

They will consist of several problems to develop by hand (or word processor) and a few computing problems (projects). They will be mostly theoretical, with about 20% of computations using MATLAB. Homeworks must be neatly written and in correct English sentences. Late homework will receive penalties. Homeworks which are overdue for more than 5 weekdays will not be accepted.

**Prerequisites**

AMSC/CMSC 466 (Introduction to Numerical Analysis) and MATH 410 (Advanced Calculus).