MATH/AMSC 673 (Fall 2003)

PARTIAL DIFFERENTIAL EQUATIONS I

Tu-Th 12:30, MTH 0405

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Prerequisites: MATH 411 or equivalent.

Objectives: This course is a first semester of a two semester sequence. It is an introduction to the classical theory of PDE. The focus will be on developing basic calculus ideas for prototype second order linear PDE, that is Laplace, heat, and wave equation, as well as first order nonlinear PDE. The second semester will treat modern methods for PDEs (distributions, functional analysis, Sobolev spaces, bounded and compact operators in Hilbert spaces).

Grading Policy

Homeworks (about 6): 45 %
Midterm: 25 %
Final: 30 %

Course Outline

1. Introduction to PDE [1 and notes]
   Derivation of basic PDEs: transport equation, conservation of mass, momentum and energy, heat and wave equations, Laplace equation, minimal surfaces, Hamilton-Jacobi
   Classification of PDE

2. Laplace's Equation [1,3]
   Dirichlet and Neumann boundary conditions
   Fundamental solution
   Mean value property
   Harnack's inequality
   Regularity of harmonic functions
   Green's functions
   Strong and weak maximum principle - Perron method
   Energy methods
   Calculus of variations, Lax-Milgram
   Finite difference and finite element methods

3. Heat Equation [1]
   Initial boundary value problems
   Fundamental solution, Duhamel's principle
   Strong and weak maximum principle
   Energy methods

4. Wave Equation [1]
   D'Alembert's formula
   Sprerical means ($n = 3$), method of descent ($n = 2$)
   Huygen's phenomenon, finite speed of propagation

Duhamel's principle
Energy method

4. First Order Nonlinear PDE [1,2]
   Characteristics: characteristic curves, systems of quasi-linear PDE, local existence
   Hamilton-Jacobi equations
   Conservation laws: scalar laws, p-system, characteristics, Riemann invariants, singularities
   Weak solutions, Rankine-Hugoniot jump conditions, nonuniqueness
   Shock waves, Lax shock conditions
   Shock and rarefaction waves for systems, Riemann problems
   Entropy conditions, viscosity solutions

Texts.

[1] L.C. Evans, *Partial Differential Equations*, Graduate Studies in Mathematics Vol 19, AMS (1998), ISBN 0-8218-0772-2.

[2] M. Renardy and R.C. Rogers, *An Introduction to Partial Differential Equations*, Texts Appl. Math. 13, Springer, 1993.

[3] D. Gilbarg and N.S. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer (1983).