AMSC 715 — Spring 2019

NUMERICAL METHODS FOR EVOLUTION PDE

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Objectives

This course covers the basic theory of finite difference methods and finite element methods for parabolic and hyperbolic partial differential equations (PDE), including first order nonlinear conservation laws. Each topic will start with a review of the corresponding PDE class.

Prerequisites

Some basic knowledge of PDE and elementary numerical analysis is recommended. The required PDE theory will be reviewed. No previous exposure to Matlab is necessary.

Textbooks

Stig Larsson and Vidar Thomée, *Partial Differential Equations with Numerical Methods*, Springer (2003).

Vidar Thomée, *Galerkin Finite Element Methods for Parabolic Problems*, Springer (2006).

Dietmar Kröner, *Numerical Schemes for Conservation Laws*, Wiley 1997.

Bradley Lucier, *Notes on Nonlinear Conservation Laws*, 1999.

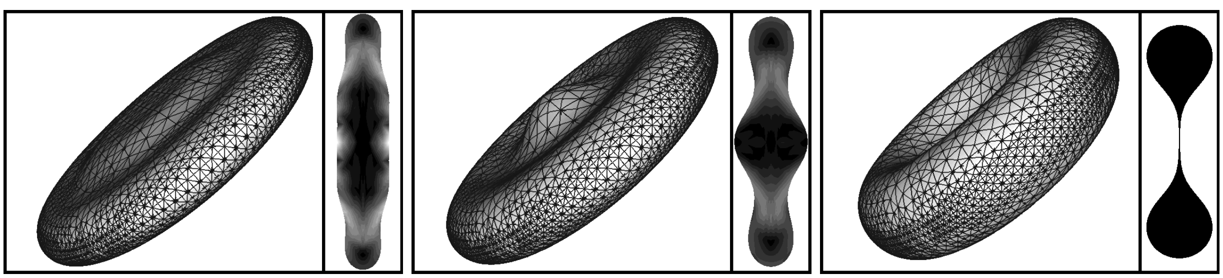

Figure 1: Evolution of a fluid biomembrane with initial axisymmetric ellipsoidal shape of aspect ratio $5 \times 5 \times 1$ and final shape similar to a red blood cell. Each frame shows the membrane mesh and a symmetry cut along a big axis. The fluid flow is quite complex, creating first a bump in the middle and next moving towards the circumference and producing a depresion in the center with flat pinching profile.

Syllabus

1. Parabolic PDE: the heat equation. Maximum principle, energy methods and Sobolev spaces, finite differences and finite element methods, stability and error estimates, applications to Navier-Stokes equations.
2. Hyperbolic PDE: the wave equation. Energy methods and conservation, finite differences and finite element methods, stability and error estimates.
3. Linear first order PDE: upwinding and monotone schemes, finite difference, finite volume, and discontinuous Galerkin methods, convergence and error estimates.
4. Nonlinear conservation laws: weak solutions and entropy conditions, Kuznetsov approximation theory, monotone finite difference methods, convergence and error estimates, Godunov methods.

Evaluation

Homeworks, both theoretical and computational (using MATLAB), with a ratio of about 75/25%.