Minicourse
ADAPTIVE FINITE ELEMENT METHODS

Ricardo H. Nochetto (www.math.umd.edu/~rhn/)

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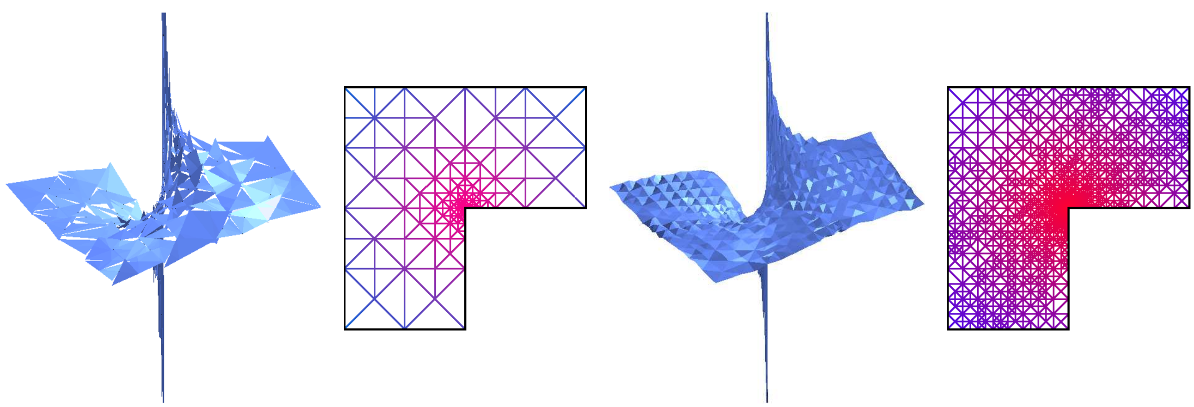
The finite element method (FEM) is now regarded as one of the most powerful tools in scientific and engineering computation. This is due to its flexibility in dealing with geometry and accuracy, particularly via local mesh refinement (adaptivity). This can in turn make complex computations feasible and greatly enhance our understanding of challenging nonlinear *multiscale* phenomena in Science, Engineering, and Finance.

This elementary minicourse will describe basic features of adaptive FEM and provide an overview of this exciting area of research. It will be followed by the graduate course AMSC 614 in FALL 04. This course will discuss in detail mathematical and computational issues of FEM, with special emphasis on adaptivity (see figure below).

Lecture 1: What is the Finite Element Method? This lecture will introduce the concepts of weak solutions of elliptic partial differential equations (PDE), Galerkin approximation, and will present the finite element method along with its basic properties. The discussion will focus mostly, but not exclusively, in 1D. No functional analysis or PDE will be required; advanced calculus will suffice.

Lecture 2: A Priori vs A Posteriori Error Estimation. This lecture will make use of Galerkin orthogonality to derive both a priori and a posteriori error estimates. They are dual counterparts and provide complementary information about the PDE and FEM. Simple a posteriori error bounds will be derived and shown to be easily computable, thereby making the assessment of approximation quality and adaptivity feasible.



Stokes flow over an L-shaped domain: Pressures and meshes for error tolerance of 5% and unstable finite element pairs (resp. DOFs): $\mathcal{P}^2$-$\mathcal{P}^1_d$ (1940), $\mathcal{P}^1$-$\mathcal{P}^1$ (4971). The oscillations do not persist under further selective refinement (nonlinear stabilization effect of adaptivity).

Lecture 3: Adaptivity. This lecture will discuss the basic ingredients for convergence of adaptive FEM, which use a posteriori error estimators. The exposition will be elementary and will be split into two parts: the first one will assess simple elliptic PDE and discuss the role of data oscillation, whereas the second part will deal with general elliptic PDE. The second part will be presented by K. Mekchay (this is part of his PhD dissertation).