AMSC 664: Final Report
Constructing Digital Elevation Models From Urban Point Cloud Data Using Wedgelets

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- Wavelets have demonstrated excellent ability at decomposing one dimensional signals. Particurally, the local support allows wavelets to very efficiently represent disconituities in the signal.
- This propertiy disapears when one talks about approximating piecewise smooth functions in $\mathbb{R}^2$ with continuous boundaries between regions using the standard tensor product wavelets.

Let f be a piecewise constant function defined on $[0, 1]^2$ with piecewise C^2 boundaries between regions.

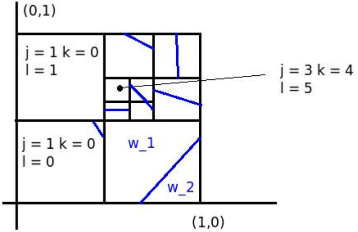
- If $\{\psi_j, k\}$ is a tensor product wavelet basis for $L_2([0, 1]^2)$ it can be shown that

$$\epsilon_N(f) = inf\{||f - \sum_{\lambda \in \Lambda} < f, \psi_\lambda > \psi_\lambda||\} = O(|\Lambda|^{-1})[3]$$

- This result is not as good as the analogous result for one dimensional signals where the approximation rate is $\epsilon_N(f) = O(|\Lambda|^{-2})$.
- Hueristically what is occouring is that the tensor product wavelet functions are detecting edges between regions as sets of isolated single points as opposed to continuous curves.

We proceed as Führ in [3]. Again let f be a piecewise constant function defined on $[0,1]^2$ with piecewise C^2 boundaries between regions. Define

- The set of dyadic squares on $[0,1]^2$ at scale j, $\mathcal{Q}_j = \{[2^{-j}k, 2^{-j}(k+1)] \times [2^{-j}l, 2^{-j}(l+1)] : 0 \leq k, l \leq 2^j\}$.
- $\mathcal{Q} = \bigcup_{j=0}^{\infty} Q_j$.
- A dyadic partition Q of $[0,1]^2$ is a tiling of $[0,1]^2$ by disjoint dyadic squares.
- A wedglet partition $W = \{(Q_j, \omega_1, \omega_2)\}$ splits each element of Q into at most two wedges ω_1, ω_2 along a suitable line.



Definition

A wedgelet segmentation is a pair (g, W) consisting of a wedge partition W and a function g which is constant on each $\omega \in W$ [3].

Definition

The wedgelet approximation of f is given by

$$min_{(g,W)} ||f - g||_2^2 + \lambda |W| [3]$$

This leads us to the following theorem.

Theorem (Approximation Properties)

Let $f : [0,1]^2 \rightarrow \mathbb{R}$ be a piecewise constant function with C^2boundary. The approximation rate for the wedgelet approximation of f is $O(|W|^{-2})$.[3]

Wedgelets were developed by Donoho in [1] for use on piecewise constant images with C^2 boundaries.

- Our justification for using wedgelets on urban terrain data is that urban data is characterized by very geometric structures (rectilinear buildings etc).
- We expand the version of wedgelets proposed by Donoho by allowing the approximating function g to be linear on each $\omega \in W$ rather than constant.
- This expansion allows us to efficiently represent structures with sloped roofs.

The elevation data is generated by a survey aircraft equipped with a LIDAR (Light Detection and Ranging) system. The data returned by the aircraft is called a point cloud.

Definition

A point cloud is a $n \times 3$ matrix with coefficients in $\mathbb{R}$ where each row represents a point in $\mathbb{R}^3$.

The data set can be thought of as a discrete random sample of the continuous elevation function.

Note that on any wedge, the approximating function g has three degrees of freedom. On a particular dyadic square, the approximating function has eight degrees of freedom.

- Three; dx_1, dy_1, z_1, determine the function $g|_{\omega_1}$.
- Three; dx_2, dy_2, z_2, determine the function $g|_{\omega_2}$.
- Two specify the location of the line in the dyadic square that separates ω_1 and ω_2.

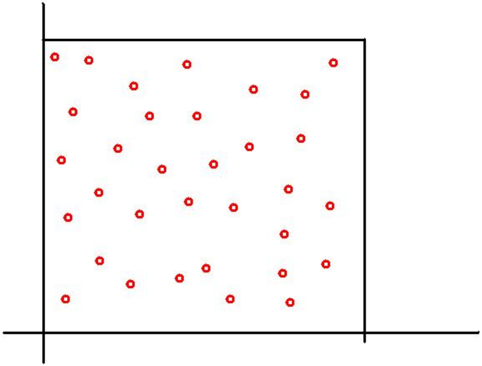
Our algorithm at each step stores the best wedge configuration in terms of L_1 error for each dyadic square in a quad-tree data structure. The approximating function $g|_{\omega_1}$ is computed using least squares minimization to the data points.

The following parameters are user set:

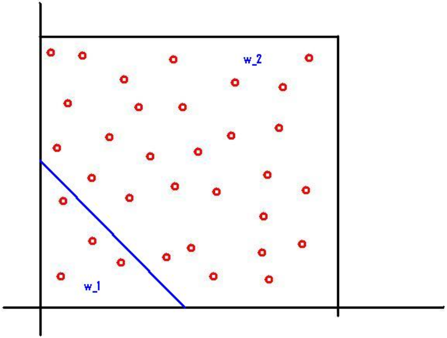
- j_{max}, The smallest scale for the dyadic squares.
- Δk The minimum line translation.
- $|\theta|$ The number of angles used in parameterizing lines.

j_{max} and Δk should be chosen small enough to capture all of the information at the scale of interest.

The data at initialization.

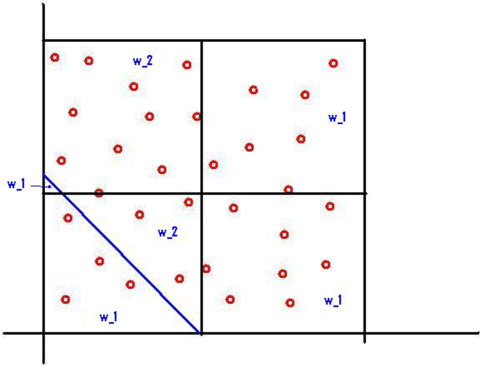


A line divides the top level into two wedges and the least squares approximation is found for ω_1 and ω_2.



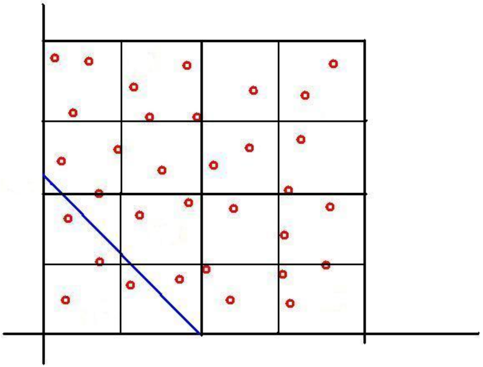
If this approximation minimizes the L_1 error over all the other approximations observed at this level so far store it at the root of the tree.

Move down one level and find the approximating function on each wedge.

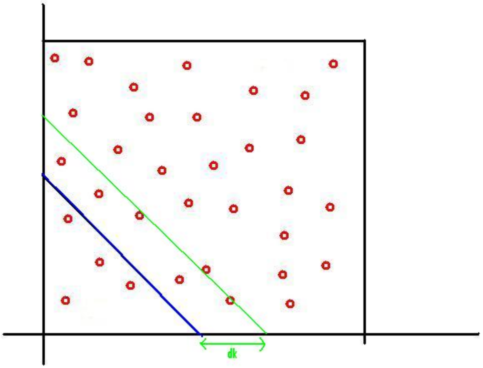


Note the existence of wedges with no data points and dyadic squares with only one wedge.

Subdivide again and repeat.

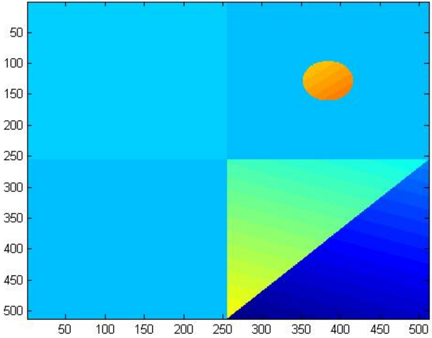


Once j_{max} has been reached return to the top level of the tree and repeat the above process with the next line in the dictionary of lines.

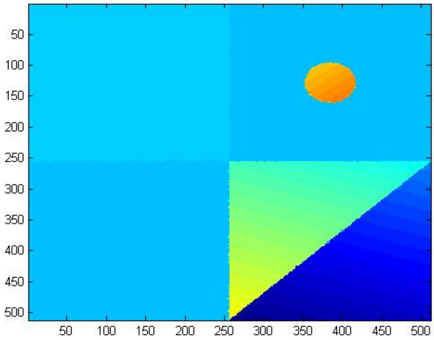


For this illustration it was assumed that we were moving breadth first through the tree. It is more computationally efficient to perform the algorithm depth first in practice and this is what we have implemented.

To verify that the algorithm was working as intended we created a simple piecewise planar function which wedgelets should be able to approximate reasonably well.

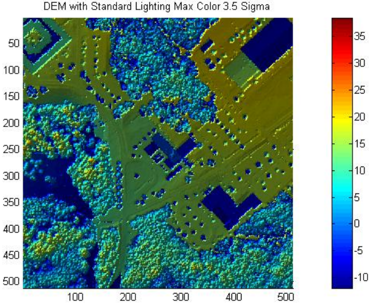


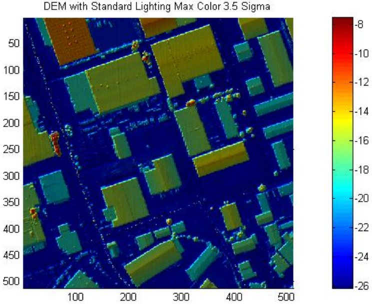
Here is the wedgelet approximation with $\lambda = 90$.



Success!! We verified that the algorithm is using three dyadic squares at the second level. The vast majority of the coefficients are being used on the circle.

To test wedgelets on real LIDAR data we used two data sources. One was taken over Ft. Belvior, VA. The other was taken over New Orleans, LA.



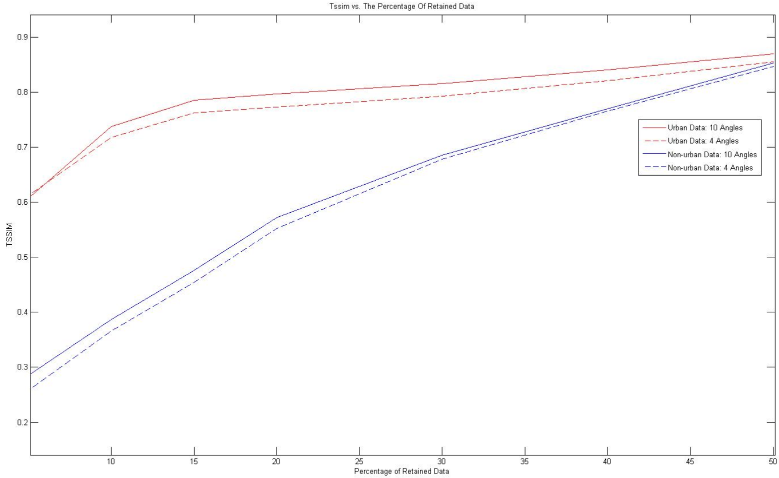


We assess the quality of the DEM's created by wedgelets by comparing them to a reference DEM using a image quality metric called TSSIM. The idea behind TSSIM was first described in [6]. TSSIM is a statistical reference measure with the following properties.

- Symmetry: $TSSSIM(X, Y) = TSSIM(Y, X)$
- Boundedness: $TSSIM(X, Y) \leq 1$
- Unique Maximum: $TSSIM(X, Y) = 1 \iff X = Y$

Empirical observation has demonstrated that TSSIM is a better measure of objective image quality than the common mathematical error norms (i.e $L^2, L^1, ...$).

The reference models are 512^2 pixels. We determine the compression rate by comparing the number of doubles that need to be stored to create a given wedgelet approximation to 512^2. The results are shown below.
In practice a TSSIM value of .75 represents a model that is sufficiently close to the reference model to be considered high quality.


Tssim vs. The Percentage Of Retained Data
TSSIM
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
10
15
20
25
30
35
40
45
50
Percentage of Retained Data
Urban Data: 10 Angles
Urban Data: 4 Angles
Non-urban Data: 10 Angles
Non-urban Data: 4 Angles

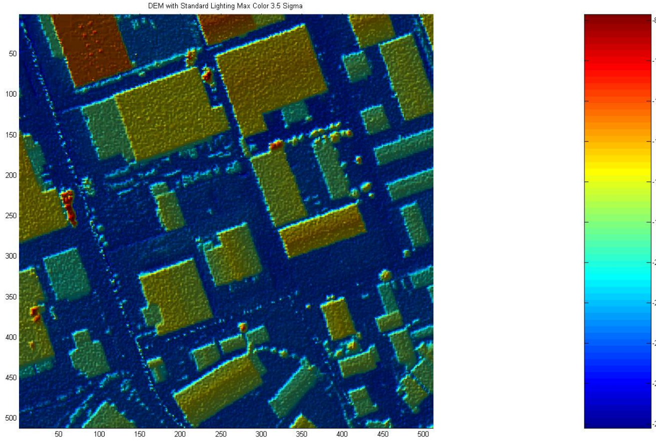
Conclusions:

- In the case of urban data wedgelets allows compression of the model to 15-20% of it's original size.
- The wedgelet representation is ineffective at representing rural data indicating that it is not suited to terrain models lacking a heavily geometric component.
- Increasing the number of angles present in the dictionary of lines improves the quality only marginally.

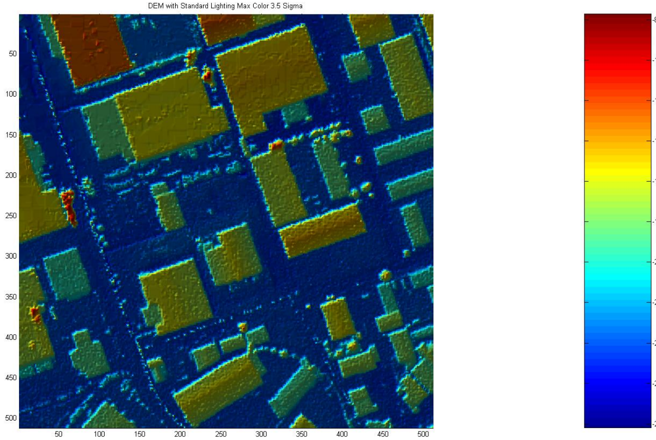
Using wedgelets to denoise images containing gaussian noise is described in [2]. The basic idea is to find a sweet spot where the wedges you use are large enough so that the interpolative process removes the noise but small enough that detail is kept.

To test this we took our the urban reference DEM and added gaussian white noise with zero mean and $\sigma = \frac{1}{3}$. Resulting wedgelet reconstructions are shown below.

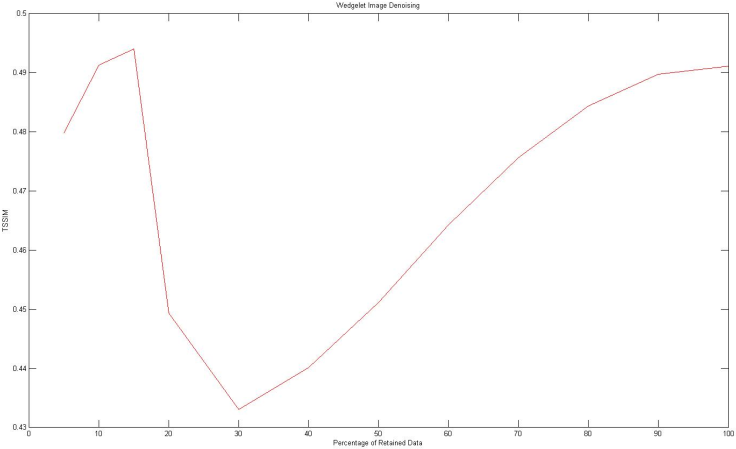
30% Retained Data



15% Retained Data



- Note the sudden disappearance of noise between the 30% and 15% levels.



Unfortunately this sudden removal of noise did not occur when the rural data was subjected to the same treatment. This indicates again that wedgelets is highly dependent on the presence of strong geometrical structures.

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