AMSC 663:Midterm Report

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- Problem Description
- Description of The Wedgelet Transform
- Implementation
- Preliminary Results
- Further Work

- Terrain survey aircraft are equipped with a LIDAR range detection system.
- These systems produce maps of terrain consisting of millions of non-uniformly sampled points in $\mathbb{R}^3$.
- This list of points is referred to as a **Point Cloud.** $P = \{(x_1, y_1, z_1), (x_2, y_2, z_2), ..., (x_n, y_n, z_n)$.

The goal of this project is to implement the wedgelet image transform on a LIDAR point cloud taken over urban terrain.

- To create a highly efficient representation of the point cloud.
- To produce a gridded image from the point cloud data.

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The wedgelet transform was described by Donoho in [1] for functions $f : [0,1]^2 \to \mathbb{R}$.

- Partition the domain into overlapping dyadic squares $Q_{i,j}$ of size 2^{-j}.
- For each $Q_{i,j}$ divide the square into two disjoint sets by drawing a line through the square.
- On each side of the line approximate f by the best fit plane through the data.
- For each $Q_{i,j}$ find the line that minimizes the error in the above step.

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The point cloud is inputed to the algorythm as an n by three matrix of floating point numbers.

$$PC = \begin{bmatrix} x_0 & y_0 & z_0 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ \vdots & \vdots & \vdots \\ x_{n-1} & y_{n-1} & z_{n-1} \end{bmatrix}$$

The matrix $P = [\vec{x}, \vec{y}, \vec{1}]$ and vector $\vec{b} = [\vec{z}]$ are initialized. A discrete set of lines is chosen.

For each line in the our set, the matrix P and vector $\vec{b}$ are divided into two parts.

$$P = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_3 & y_3 & 1 \\ \vdots & \vdots & \vdots \end{bmatrix} P_1 = \begin{bmatrix} x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \\ x_7 & y_7 & 1 \\ \vdots & \vdots & \vdots \end{bmatrix} P_2 = \begin{bmatrix} x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_5 & y_5 & 1 \\ x_6 & y_6 & 1 \\ \vdots & \vdots & \vdots \end{bmatrix}$$

The vector $\vec{b}$ is divided into $\vec{b_1}$ and $\vec{b_2}$ in the same manner.

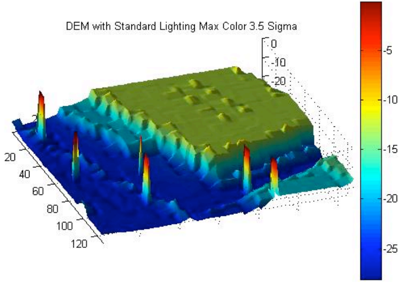
- For each of these two matrices the least squares solution to the system $P[dx, dy, z_0]^t = \vec{b}$ is computed.
- The plane $f(x, y) = dx \cdot x + dy \cdot y + z_0$ is the best fit plane for the data on each side of the line.
- The residuals are computed and stored.
- If the sum of the residuals for this line is the smallest yet found for a given square, then the wedge parameterized by $(error, \theta, k, dx_1, dy_1, z_1, dx_2, dy_2, z_2)$ is stored.

If either matrix is underdetermined, (i.e contains zero, one, or two points) we handle it as follows,

- Zero Points: The wedge is disregarded if the line passes through the image space.
- One Point: The wedge is set to a constant agreeing with the data point.
- Two Points: The wedge is set to zero in the undetermined dimension and the system is solved using this information.

An additional special case arises when either matrix is (nearly) rank deficient.

Instability Using the QR Factorization



- If either matrix contains fewer than 20 points then the SVD is used.
- Small singular values are zeroed and the least squares problem is only solved in terms of the large singular values.
- This approach resolves the stability issue that often arises when the matrix has few points and is nearly rank deficient.

- We want to be able to control the coarsness of our image reconstructed from the wedgelet transform.
- A coarser image will have lower quality but will require fewer wedges to be stored.
- For fixed $\lambda \geq 0$, Donoho defined the optimal wedgelet partition of an image to be the minimizer of the functional,

$$F(|W|, \lambda) = ||f - g|| + \lambda |W|$$

- λ acts to penalize wedgelet approximations using large numbers of wedges.

Our tree pruning algorithm works by starting at the lowest level of the tree.

- For each set of four related leaves determine if using the parent instead of the four individual children decrease the value of the functional.
- Look for the set of four child nodes where the improvement is the greatest and swap the parent for the four children.
- Continuously apply this criterion until no improvement can be made.

Construction of an image from the chosen wedgelet partition proceeds as follows.

- Choose the size of the image in pixels.
- Dyadically partition the image space in the same manner as the point cloud.
- For a given pixel identify which active wedge it is contained in.
- Set a value for that pixel using the information encoded in the wedgelet coefficient for that node.

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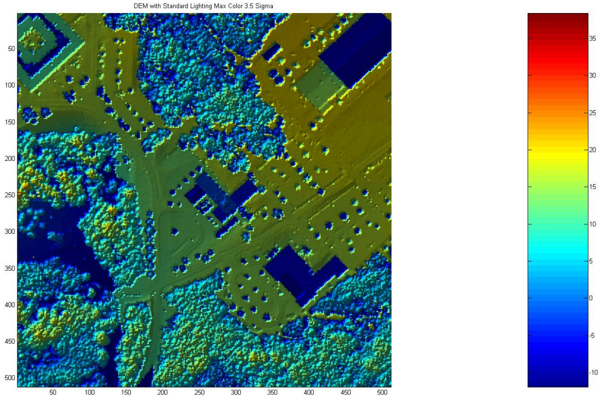
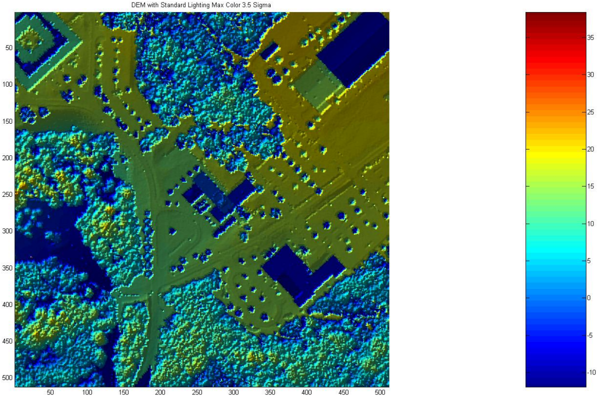
Sample point cloud came from the Army Corps of Engineers TEC division. The average sampling density was $2\frac{sample}{m^2}$. The following images are displayed at a scale $1\ pixel = 1m^2$.

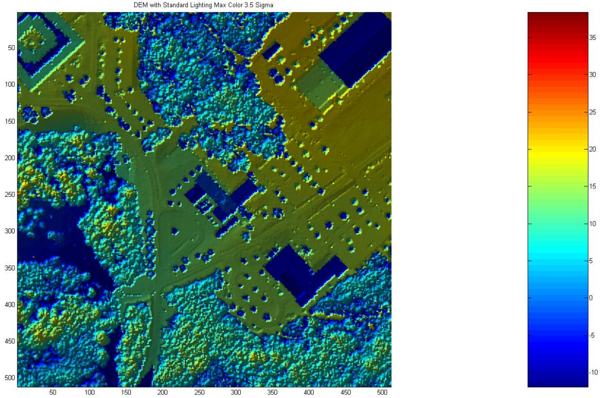
Image Reconstructed from Point Cloud using Matlab's Gridding



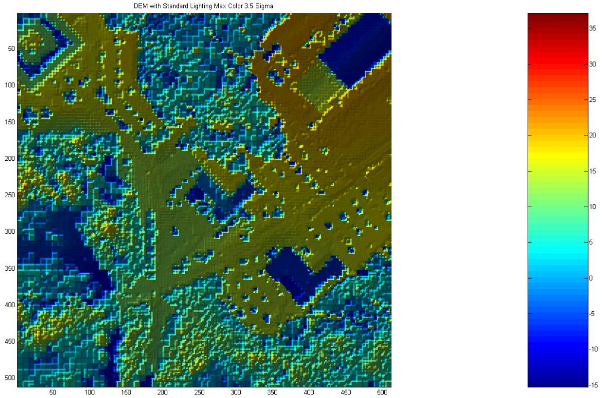
Using Wedgelets $\lambda = 1$



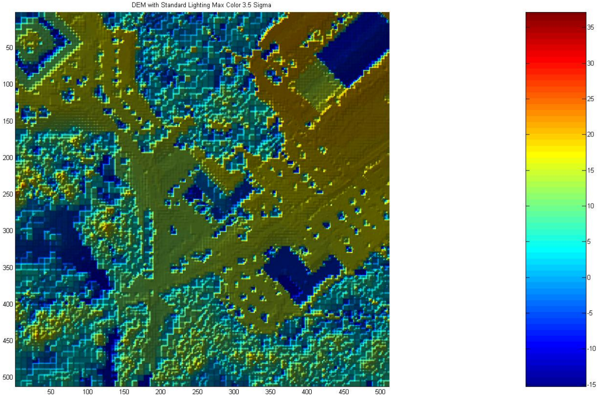
Using Wedgelets $\lambda = 5$



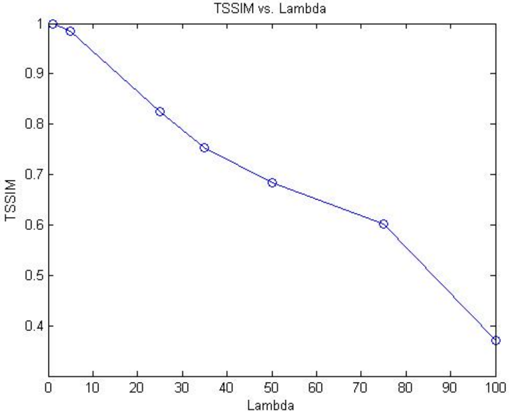
Using Wedgelets $\lambda = 75$



Using Wedgelets $\lambda = 100$



Structural Similarity Index (TSSIM) Between Original and Reconstructions



January

- Implement de-noising algorithms. [4]
- Assess Compression Rates.
- Parallelize wedgelets algorithm.

February

- Assess effectiveness of de-noising algorithms.
- Implement Moenning and Dodgson's point cloud reduction algorithm. [3]
- Assess effectiveness.

March

- Develop GUI front end.
- Begin Final Report.

April

- Complete Final Report First Draft.

May

- Present Results.
- Deliver Final Report.

- David L. Donoho *Wedgelets: Nearly Minimax Estimation of Edges*, The Annals of Statistics, Vol. 27, No. 3. (Jun., 1999), pp. 859-897.
- Laurent Demaret, Felix Friedrich, Hartmut Fhr, Tomasz Szygowski, *Multiscale Wedgelet Denoising Algorithms*, Proceedings of SPIE, San Diego, August 2005, Wavelets XI, Vol. 5914, X1-12
- Moenning C., Dodgson N. A.: *A New Point Cloud Simplification Algorithm*. Proceedings 3rd IASTED Conference on Visualization, Imaging and Image Processing (2003).
- S. Sotoodeh , *Outlier Detection In Laser Scanner Point Clouds*, IAPRS Volume XXXVI, Part 5, Dresden 25-27 September 2006