NUMERICAL SIMULATION OF DYNAMIC STALL

Debojyoti Ghosh

Adviser: Dr. James Baeder
Alfred Gessow Rotorcraft Center
Department of Aerospace Engineering

Proposal

- To study the **Dynamic Stalling** of rotor blade cross-sections
- **Unsteady Aerodynamics**:
  - Time varying angle of attack and free-stream velocities.
  - Affects the lift, drag and pitching moment of the rotor
- Numerical Simulation:
  - Solution of the **Navier Stokes** equations with an appropriate turbulence model
  - “First Step” → Solve the **Euler Equations** (inviscid aerodynamics)

Introduction

- **Airfoil**:

    Wing / Rotor cross section – basic 2D lifting surface

- **Lift** and **Drag** are functions of **angle of attack**, **free-stream fluid velocity** and **shape** of the airfoil
- Higher velocities on upper surface create pressure difference resulting in aerodynamic forces
- **Inviscid** flow over airfoil → only pressure forces, no shear stresses on the surface
    - **Inviscid drag** less than actual drag

Inviscid Compressible Aerodynamics

- **Governing Equations:** Euler Equations
  - Conservation of **Mass, Momentum and Energy**
- Obtained from the Navier Stokes equations by neglecting viscosity and heat conduction
- Importance: (High Speed Flows)
  - Flow around any solid body = viscous “boundary layer” + outer flow
  - Flow away from the surface can be approximated as inviscid flow (negligible cross-derivatives of fluid velocity)

Finite Volume (FV) Formulation

- Discretizations based the **Integral form** of the governing equation

$$\int_{V} \frac{\partial \mathbf{u}}{\partial t} dV + \int_{\partial V} \mathbf{F}.\hat{\mathbf{n}} dS = 0$$

- Also called as **"Conservation form"** since **u** is a conserved variable
- Does not assume smooth solutions (unlike differential form)

  → More appropriate for hyperbolic systems with discontinuous solutions

Governing Equations

- **Conservation form** of the **Euler Equations**

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla.\mathbf{F} = 0; \mathbf{F(u)} = \mathbf{f(u)}\hat{\mathbf{i}} + \mathbf{g(u)}\hat{\mathbf{j}}$$

$$\mathbf{u} = \begin{bmatrix} \rho \\ \rho \mathrm{u} \\ \rho \mathrm{v} \\ E \end{bmatrix}, \mathbf{f(u)} = \begin{bmatrix} \rho \mathrm{u} \\ \rho \mathrm{u}^2 + \mathrm{p} \\ \rho \mathrm{uv} \\ (\mathrm{E} + \mathrm{p})\mathrm{u} \end{bmatrix}, \mathbf{g(u)} = \begin{bmatrix} \rho \mathrm{v} \\ \rho \mathrm{uv} \\ \rho \mathrm{v}^2 + \mathrm{p} \\ (\mathrm{E} + \mathrm{p})\mathrm{v} \end{bmatrix}$$

ρ - Density, (u, v) – Velocity components,
p - Pressure, E – Internal Energy

Equation of State

$$\mathrm{E} = \frac{\mathrm{p}}{\gamma - 1} + \frac{1}{2}\rho(\mathrm{u}^2 + \mathrm{v}^2)$$

Numerical Scheme

- **Semi-discrete equation** using the FV formulation:

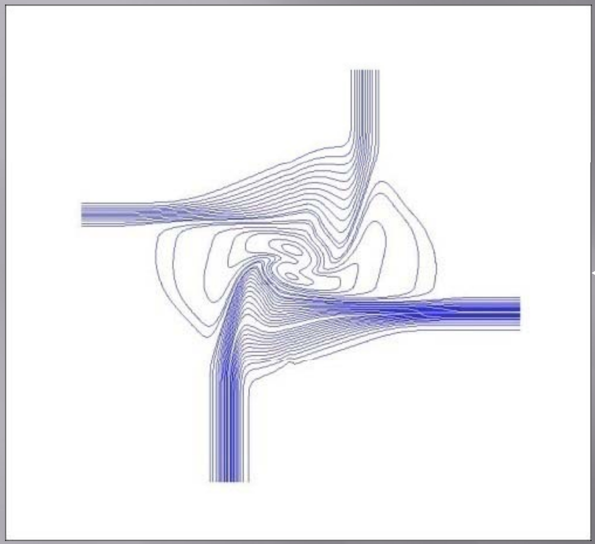
$$\frac{d\mathbf{u}_{ij}}{dt} + \sum_{faces} \mathbf{F}.\hat{\mathbf{n}}\, dS = 0 \Rightarrow \frac{d\mathbf{u}_{ij}}{dt} = \mathbf{Res}(i, j)$$

- **Flux Computation** normal to cell interfaces
  - Upwinded to account for wave nature of the solution
  - **Essentially Non Oscillatory Schemes** (2^{nd}, 3^{rd} order)
- Time Marching using Total Variation Diminishing Runge Kutta (2^{nd}, 3^{rd} order) schemes

Validation

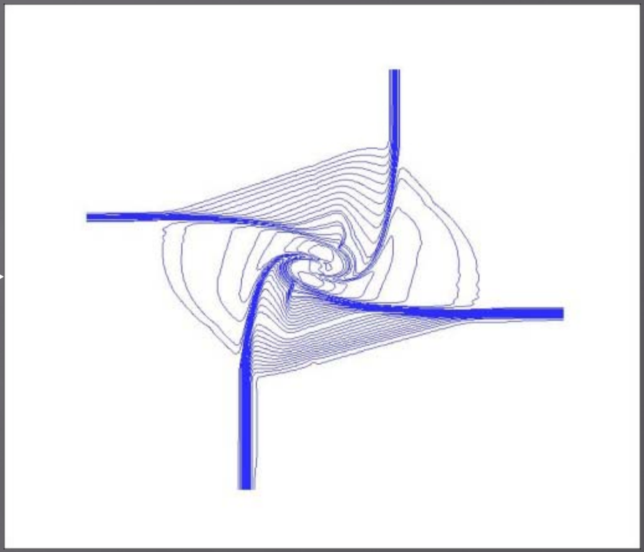
- **2D Riemann Problems on Cartesian grids**
  - Discontinuous initial data on a square domain
  - Unsteady problems
- **Mach 2.9 Oblique Shock Reflection problem**
  - Oblique shock wave at 30^o reflects off a flat wall
- **Supersonic flow on 15^o compression ramp**
  - Compression ramp causes an oblique shock followed by an expansion
- **Inviscid flow around the NACA0012 airfoil**
  - Subsonic and Transonic cases studied and validated
  - Results validated with UM TURNS code
    - developed and used by the Rotorcraft Center
    - Implicit time – stepping with MUSCL-type reconstruction

2D Riemann Problems



1st Order

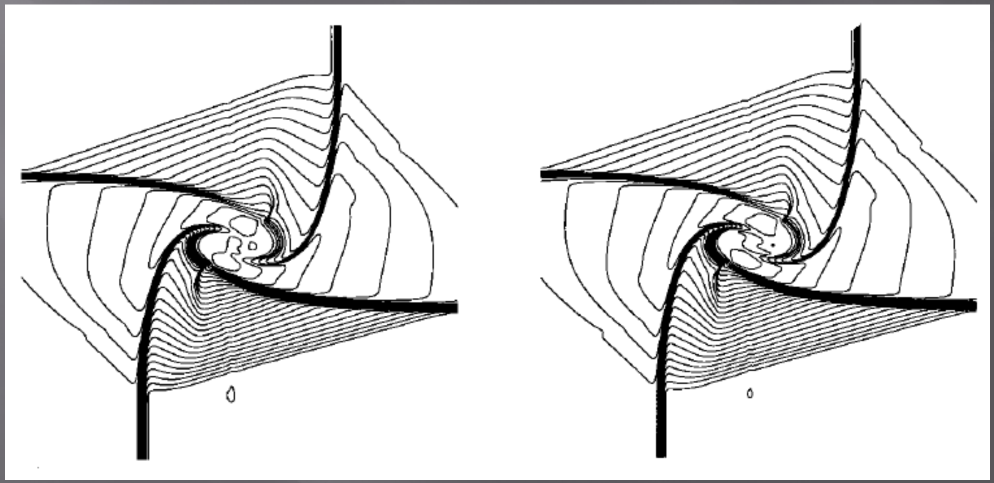
My results



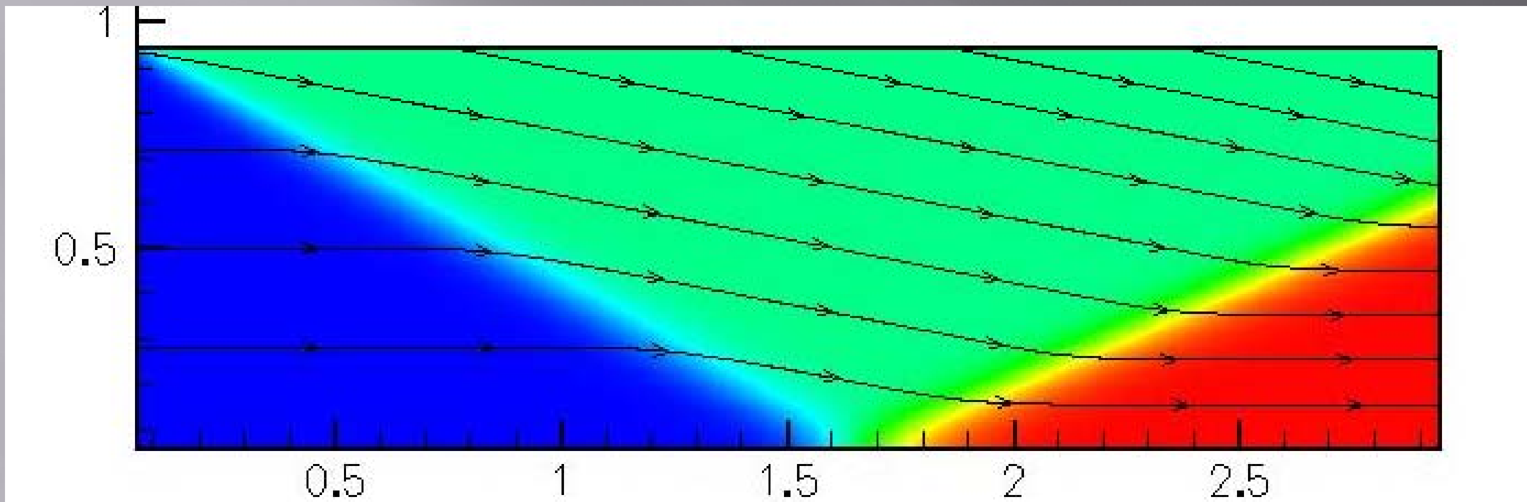
3rd Order

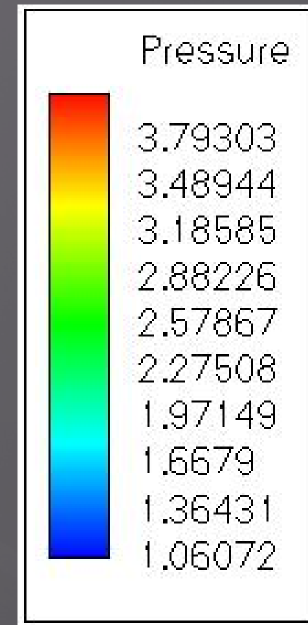
"Configuration 6" (Density)

Results from Kurganov-Tadmor



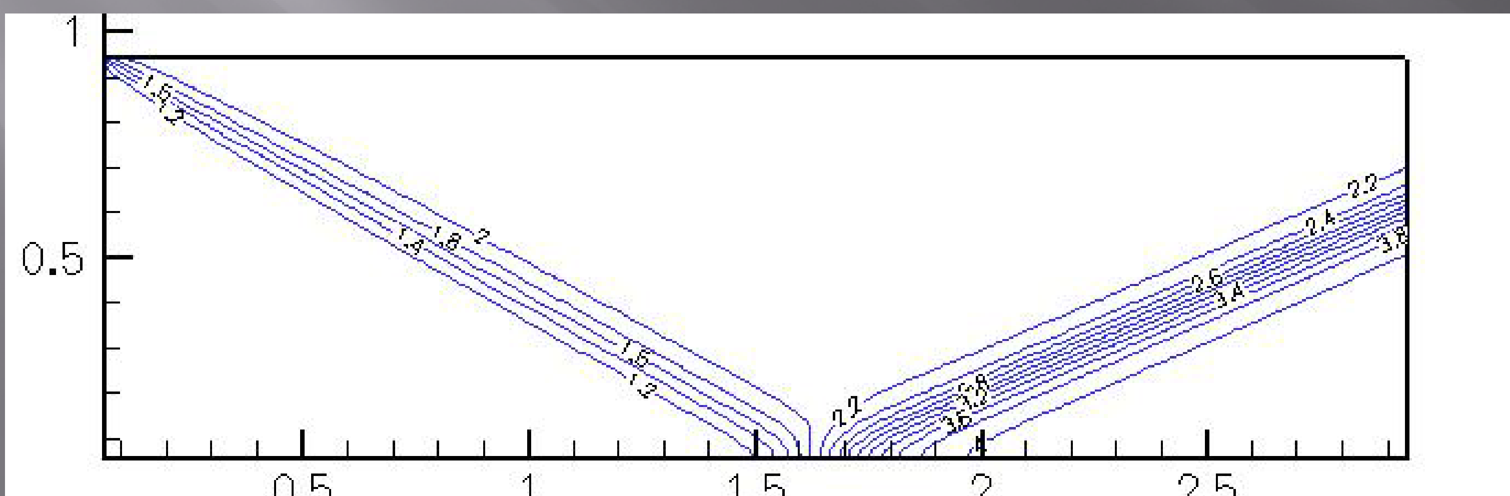
Oblique Shock Reflection





Oblique Shock Pressure Contours and Streamlines

Exact Solution obtained through Oblique Shock relations

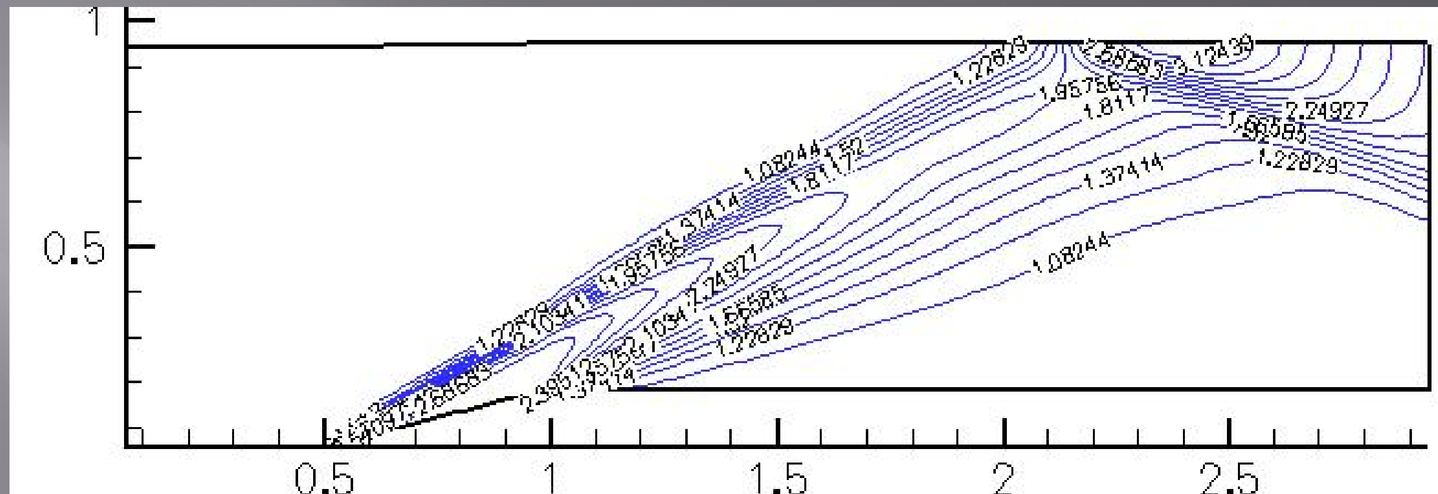


3rd order ENO +
3rd order TVD
Runge Kutta

Mach 2.9 Inflow

Flow through Compression Ramp

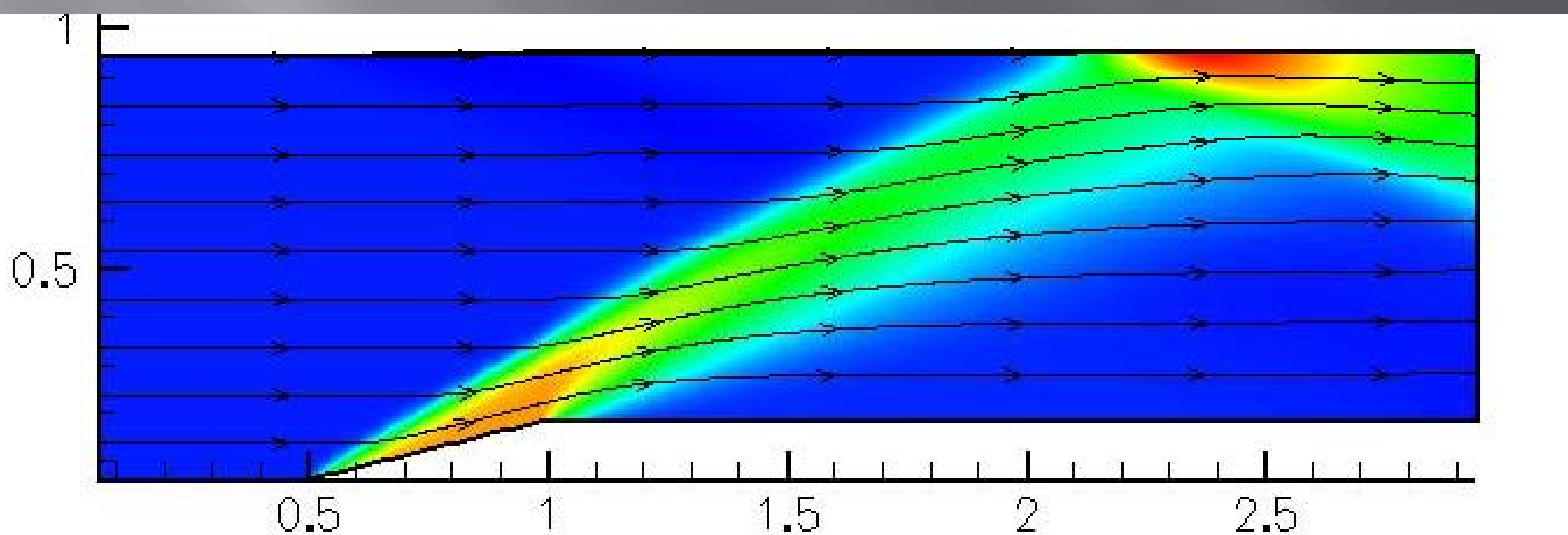
2nd order ENO +
2nd order TVD
Runge Kutta

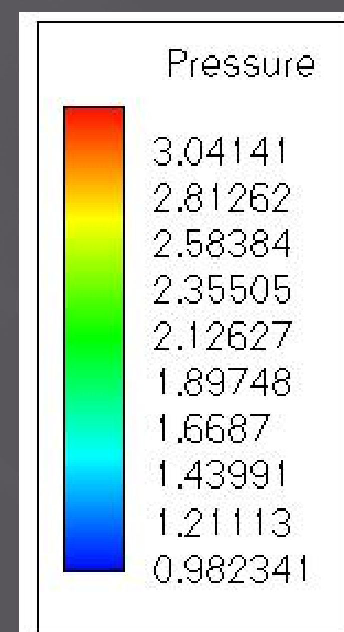


Mach 3.3 Inflow

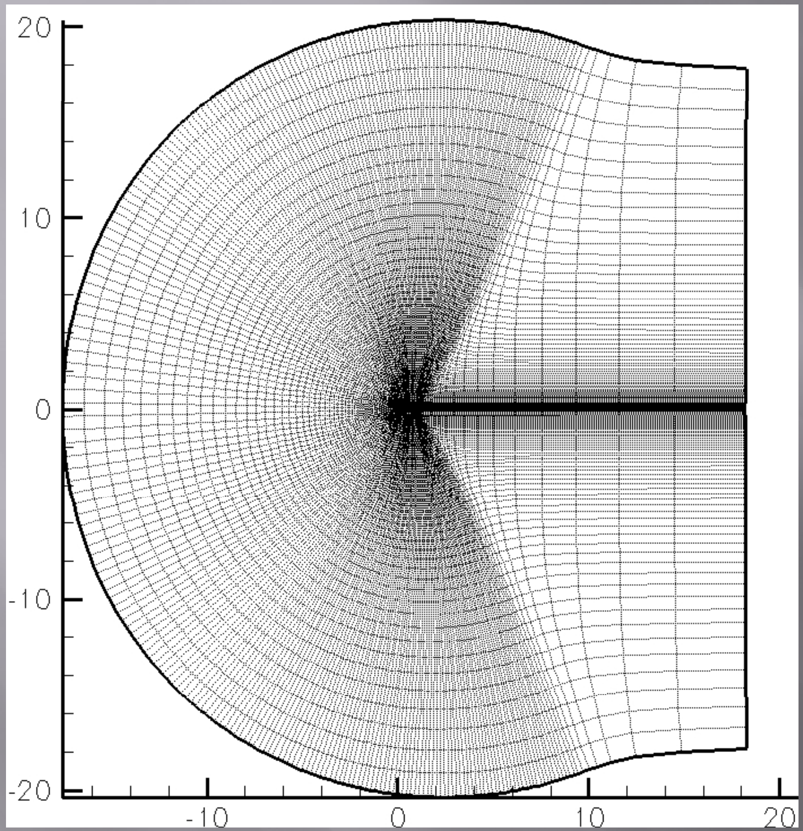
Compression Ramp Pressure Contours and Streamlines

Solution validated with exact solution obtained from oblique shock relations and Prandtl-Myer expansion fan relations





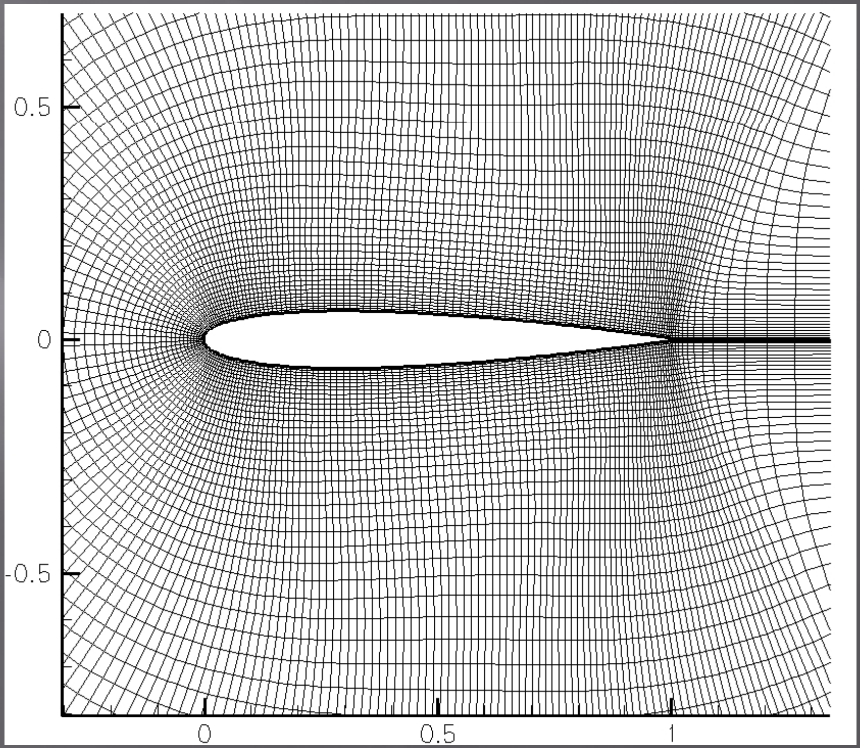
Airfoil Computations – Domain



C-Type Structured Mesh with outside boundary 20 chords away

Freestream boundary conditions on outer boundary

Magnified view of mesh around airfoil (unit chord)



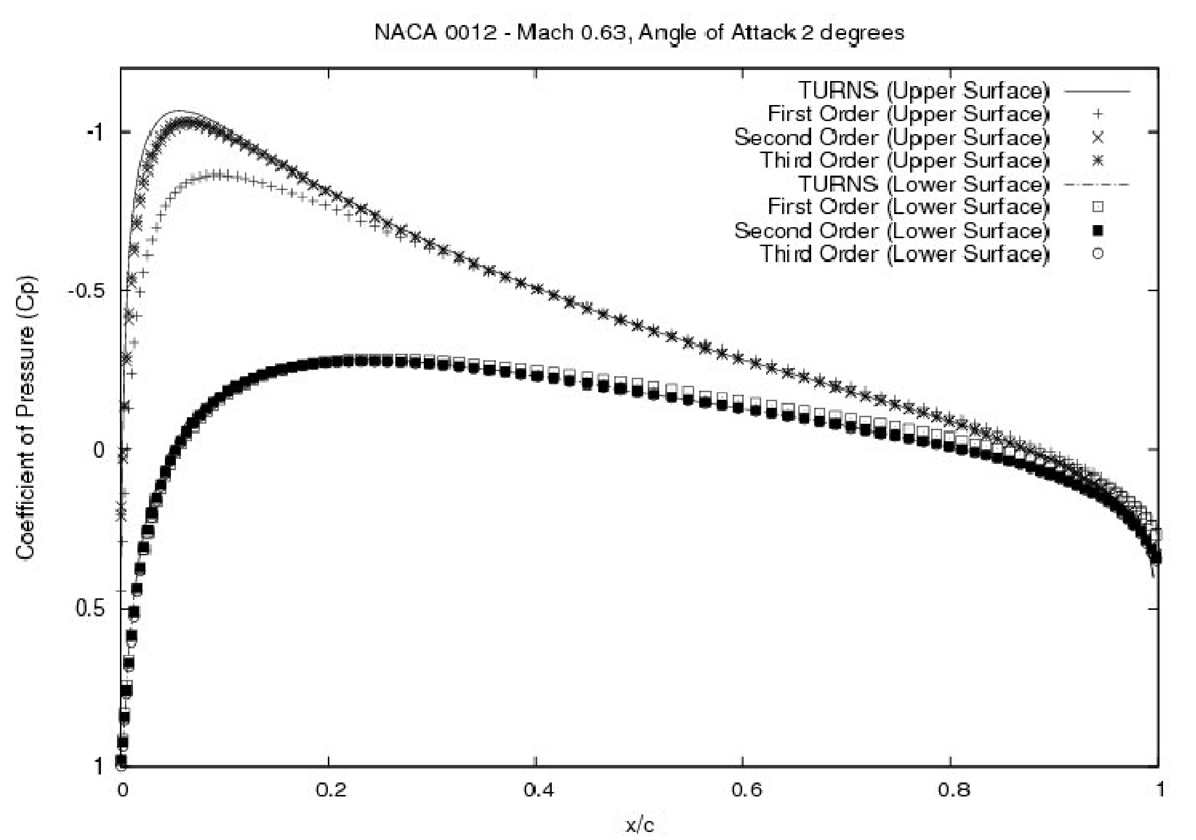
Curved Wall Boundary Conditions at Airfoil Surface

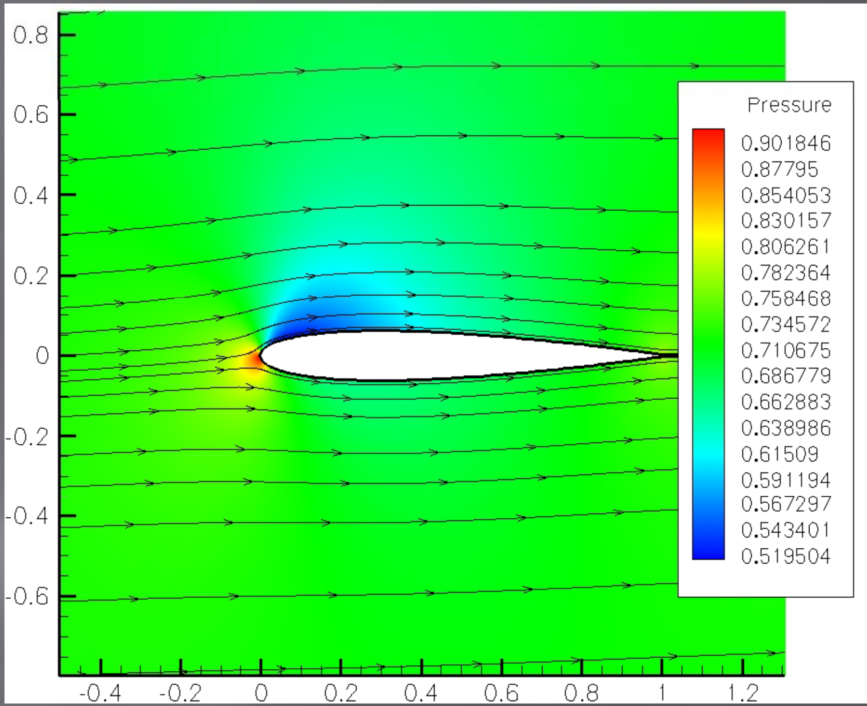
NACA0012 Subsonic

- Coefficient of Pressure $$C_p = \frac{p - p_\infty}{\rho_\infty u_\infty^2/2}$$

- Results validated with TURNS code

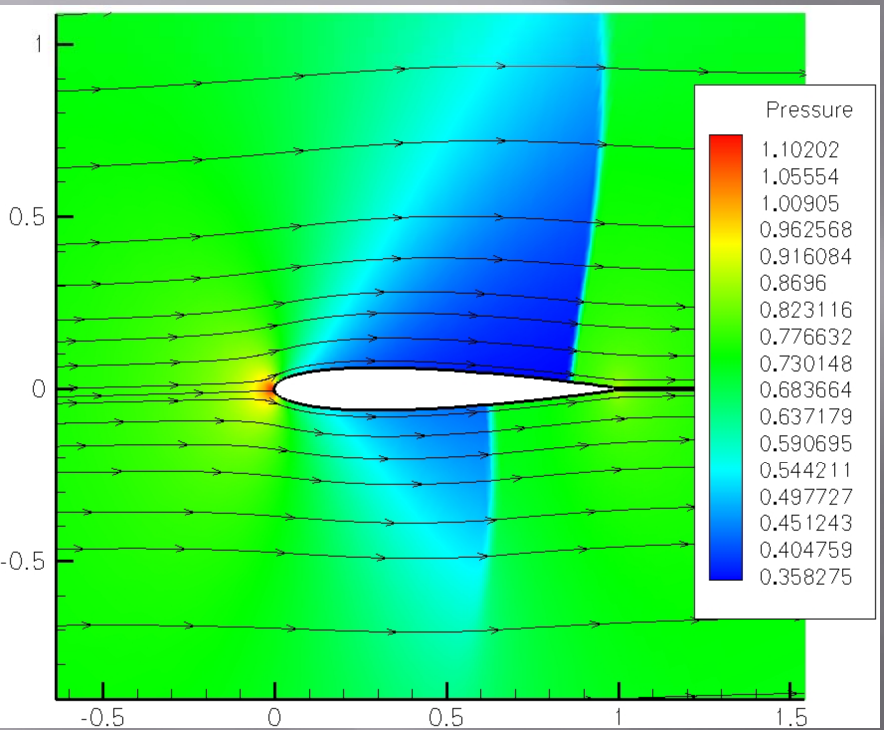
Higher order schemes capture suction peak better than 1st order





Pressure and streamlines around the airfoil at Mach 0.63 and 2 degrees angle of attack

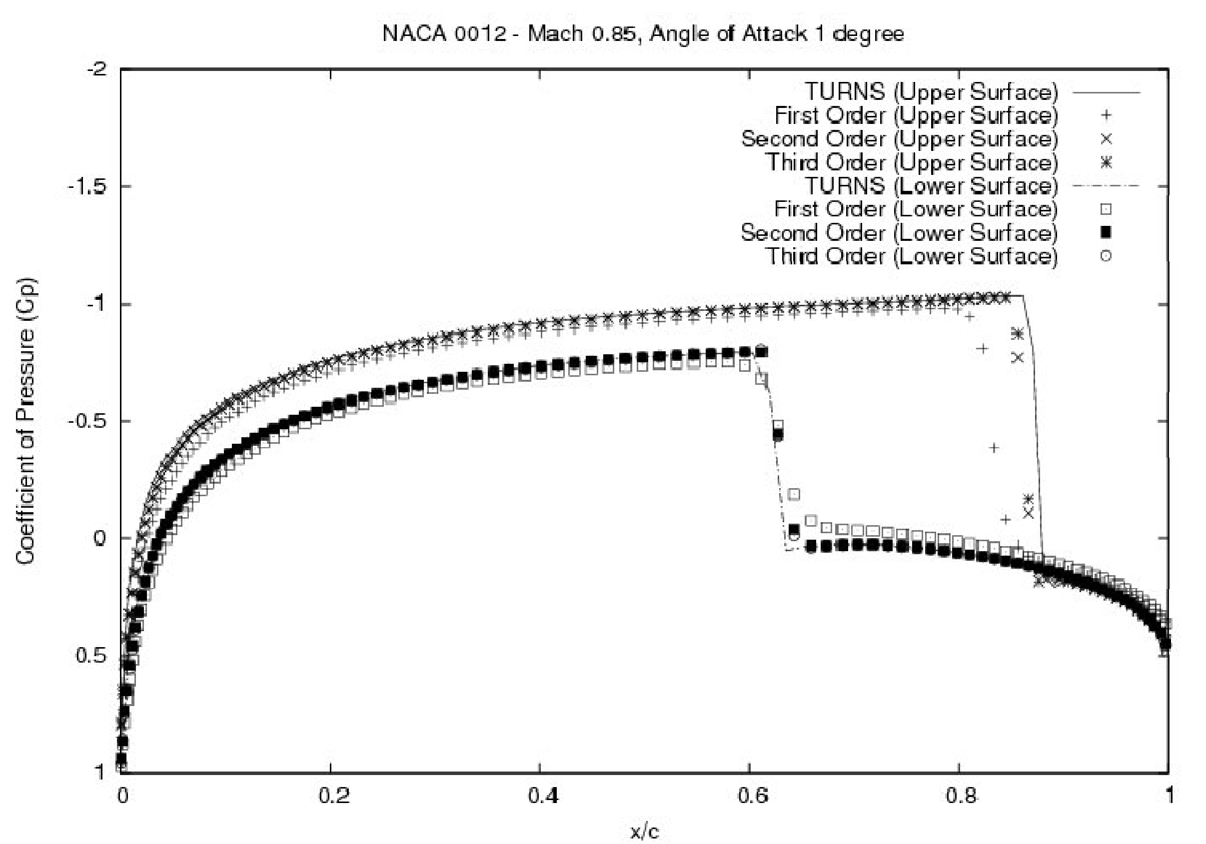
NACA0012 Transonic



Results validated with TURNS code

Higher order schemes show better shock resolution than 1st order

Pressure and streamlines around the airfoil at Mach 0.85 and 1 degrees angle of attack



Conclusions

- 2D Euler code validated for various cases
- Next steps:
  - Incorporating viscosity terms in the code to make in a Navier Stokes solver
  - Validation of the Navier Stokes solver on 2D problems (Cartesian and non – Cartesian)
  - Incorporating a turbulence model
- Timeline → Running slightly late but will make up
  - Finished with building and validating 2D Euler code
  - Started reading up on solution to Navier Stokes equations
  - Have coded in the viscous terms for the Navier Stokes solver (will start validating soon)

END