Numerical Simulation of Dynamic Stall

AMSC 663-664 Project

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Introduction

- Aim
  - To study the dynamic stalling of rotorcraft blade sections
- Dynamic Stall
  - Major factor in helicopter performance
  - Involves unsteady aerodynamics (time varying angle of attack and incoming flow velocity)
- Numerical Solution
  - Navier Stokes equations w/ turbulence model

Flow around Airfoil

- Airfoil: 2D cross-section of wing/rotor
  - Exhibits basic characteristics of flow around wings and other lifting surfaces
  - Provides an estimate of the lift and drag for actual flows
- Numerical Solution of flow around airfoils
  - 2D Euler or Navier Stokes equations
  - To study nature of flow over wings, away from edges
  - Step towards building a 3D flow solver

Inviscid Flows

- Governed by the Euler Equations
  - Based on the conservation of mass, momentum and energy
  - Neglects viscosity and heat conduction
- Relevance
  - High speed flows, away from solid surfaces
  - Negligible cross-derivatives of flow velocity
  - Provides a good estimate of lift but not drag

Viscous Flows

- Governed by the Navier Stokes equations
  - Effect of viscous forces and heat conduction on the momentum and energy equations
- Required to compute drag and flow separation
  - Drag = pressure drag + shear forces
  - Flow separation at high angles of attack (pockets of reverse flows)
- Computationally more expensive
  - Requires very fine mesh spacing near body surface to capture boundary layer

Governing Equations

Euler Equations (Inviscid) $$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{f}(\mathbf{u})}{\partial x} + \frac{\partial \mathbf{g}(\mathbf{u})}{\partial y} = 0$$

Navier Stokes Equations $$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{f}(\mathbf{u})}{\partial x} + \frac{\partial \mathbf{g}(\mathbf{u})}{\partial y} = \frac{\partial \mathbf{f}_v(\mathbf{u})}{\partial x} + \frac{\partial \mathbf{g}_v(\mathbf{u})}{\partial y}$$

$$\mathbf{u} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ E \end{bmatrix}, \mathbf{f}(\mathbf{u}) = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ (E+p)u \end{bmatrix}, \mathbf{g}(\mathbf{u}) = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ (E+p)v \end{bmatrix}, \mathbf{f}_v(\mathbf{u}) = \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ k\partial_x T + u\tau_{xx} + v\tau_{xy} \end{bmatrix}, \mathbf{g}_v(\mathbf{u}) = \begin{bmatrix} 0 \\ \tau_{yx} \\ \tau_{yy} \\ k\partial_y T + u\tau_{yx} + v\tau_{yy} \end{bmatrix}$$

$$\tau_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y}; \tau_{yy} = \lambda \frac{\partial u}{\partial x} + (\lambda + 2\mu)\frac{\partial v}{\partial y}; \tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

Constitutive Relations

- Equation of State $$E = \frac{p}{\gamma - 1} + \frac{1}{2}\rho(u^2 + v^2)$$
- Ideal Gas Law $$p = \rho RT$$
- Fluid Properties:
  - μ Fluid Viscosity Coefficient
  - λ Bulk Viscosity Coefficient
  - k Thermal Conductivity
  - R Universal Gas Constant
  - γ Ratio of Heat Capacities

Finite Volume Formulation

- Based on the Integral Form of the governing equations

$$\int_{V} \frac{\partial \mathbf{u}}{\partial t} dV + \int_{\partial V} \mathbf{F}.\widehat{\mathbf{n}} dS = 0$$

$$\mathbf{F}(\mathbf{u}) = [\mathbf{f}(\mathbf{u}) - \mathbf{f}_v(\mathbf{u})]\hat{\mathbf{i}} + [\mathbf{g}(\mathbf{u}) - \mathbf{g}_v(\mathbf{u})]\hat{\mathbf{j}}$$

- Semi-discrete equation

$$\frac{d\mathbf{u}_{ij}}{dt} + \sum_{faces} \mathbf{F}.\hat{\mathbf{n}} dS = 0 \Rightarrow \frac{d\mathbf{u}_{ij}}{dt} = \mathbf{Res}(i, j)$$

Inviscid Flow Computations

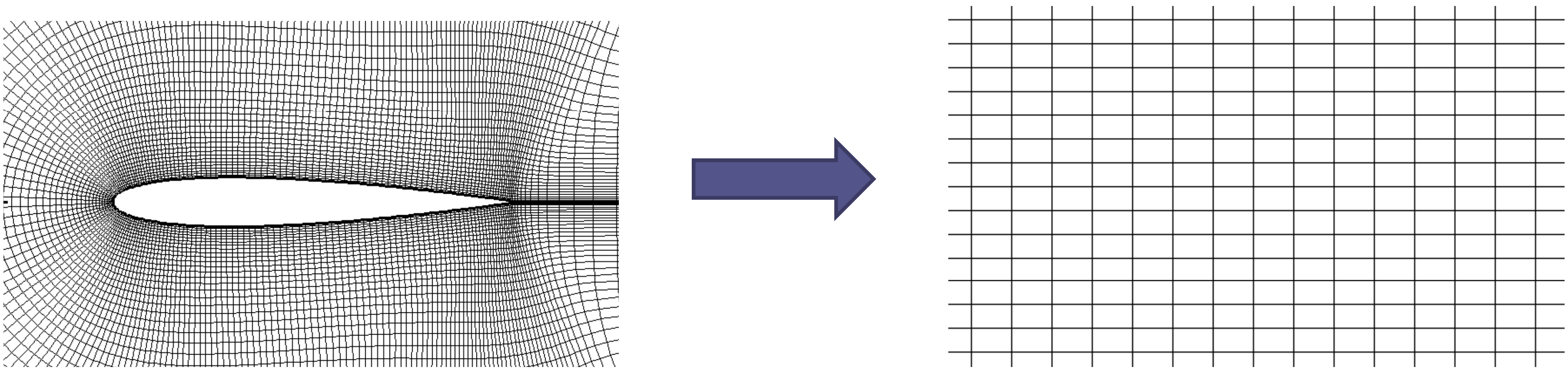
- Upwinded flux reconstruction at the interfaces
  - Characteristics based upwinding (solution composed of acoustic and entropy waves)
  - 1st Order Roe Scheme
  - 2nd, 3rd Order Essentially Non-Oscillatory schemes
- Explicit Time Marching
  - Runge Kutta family of ODE solvers
  - 2nd and 3rd order Total Variation Diminishing RK schemes

Viscous Flow Computations

- Explicit time marching unsuitable
  - Extremely small mesh spacing near the surface results in a very restrictive stability limit for time step size
- Implicit Time Stepping: Backward Euler
  - Unconditionally stable
  - Computationally more intensive per iteration (solution of a system of equations)
  - Non-linear system of equations requires linearization in time for each time step
  - Time step size limited by accuracy considerations

Curvilinear Coordinates

- Transformation of the governing equations from Cartesian coordinates (x, y) to curvilinear coordinates (ξ, η)



- Metrics of transformations numerically computed

$$x_\xi,\ y_\xi,\ x_\eta,\ y_\eta \quad \leftrightarrow \quad \xi_x,\ \xi_y,\ \eta_x,\ \eta_y$$

Numerical Scheme

- Governing Equation in curvilinear coordinates

$$\frac{\partial \hat{\mathbf{u}}}{\partial t} + \frac{\partial \hat{\mathbf{f}}(\hat{\mathbf{u}})}{\partial \xi} + \frac{\partial \hat{\mathbf{g}}(\hat{\mathbf{u}})}{\partial \eta} = \frac{\partial \hat{\mathbf{f}}_v(\hat{\mathbf{u}})}{\partial \xi} + \frac{\partial \hat{\mathbf{g}}_v(\hat{\mathbf{u}})}{\partial \eta}$$

- Linearization of flux with respect to time

$$f(u(t + \Delta t)) = f(u(t)) + A\Delta u + O(\Delta t^2);\ A = \frac{\partial f}{\partial u}$$

- Implicit formulation (h – Time step)

$$[I + h\partial_\xi A + h\partial_\eta B]\Delta u = h[\partial_\xi \hat{f} + \partial_\eta \hat{g}];\ A = \frac{\partial f}{\partial u},\ B = \frac{\partial g}{\partial u}$$

$$\Rightarrow [I + h\partial_\xi A][I + h\partial_\eta B]\Delta u = h[\partial_\xi \hat{f} + \partial_\eta \hat{g}] \qquad \text{(ADI)}$$

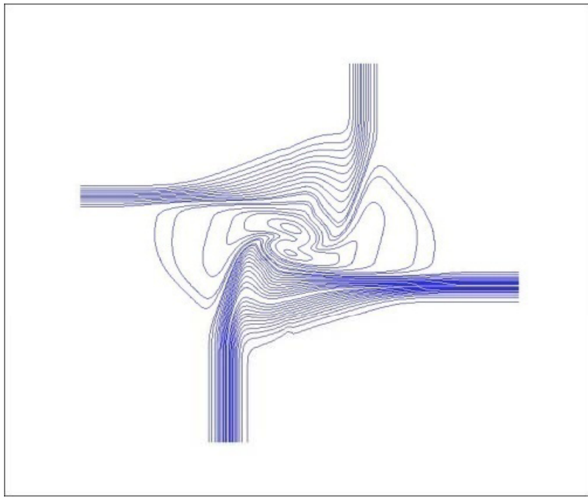
Numerical Scheme

- Right-hand side computed using finite difference/finite volume formulations
  - Convective flux computed as before
  - Dissipative terms computed using second order central differencing
- Left-hand side
  - Banded penta-diagonal system
  - ADI approximation: two banded tri-diagonal systems
  - $\partial_\xi(A\Delta u)$, $\partial_\eta(B\Delta u)$ computed using 1^{st} order upwind finite differences

Validation

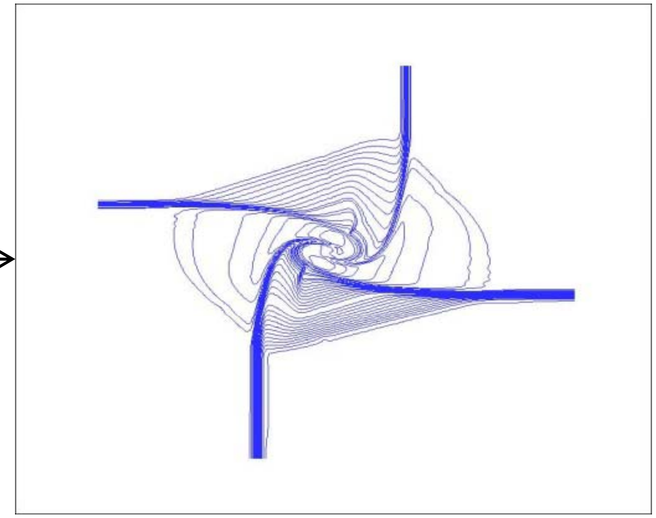
- Cartesian Meshes
  - 2D Riemann Problems (Inviscid)
  - Oblique Shock Reflection (Inviscid)
  - Couette Flow (Viscous)
- Airfoil computations
  - NACA0012 Subsonic (Inviscid)
  - NACA0012 Transonic (Inviscid)
  - RAE2822 Transonic (Viscous)
  - NACA0012 High Angle of Attack (Viscous)

2D Riemann Problems



1st Order

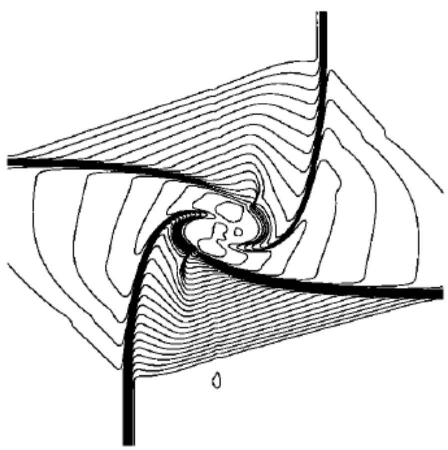
My results

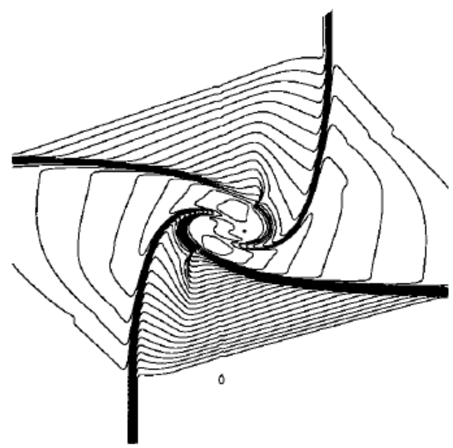


3rd Order

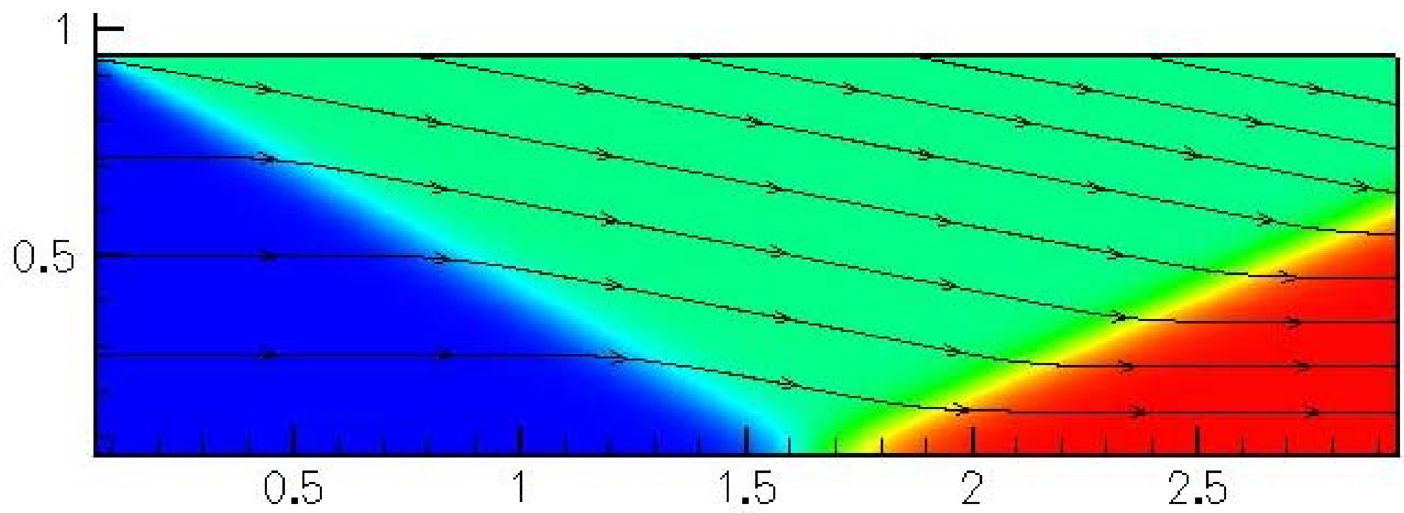
“Configuration 6” (Density)

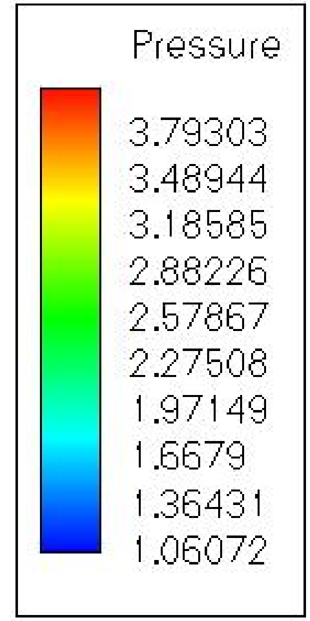
Results from Kurganov-Tadmor





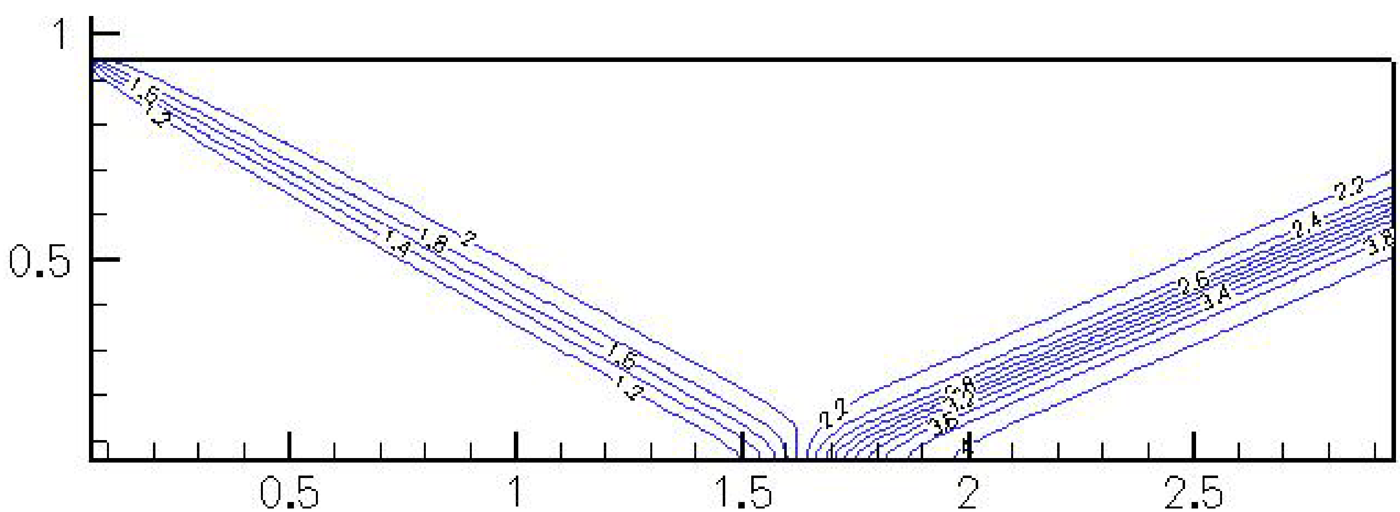
Oblique Shock Reflection





Oblique Shock Pressure Contours and Streamlines

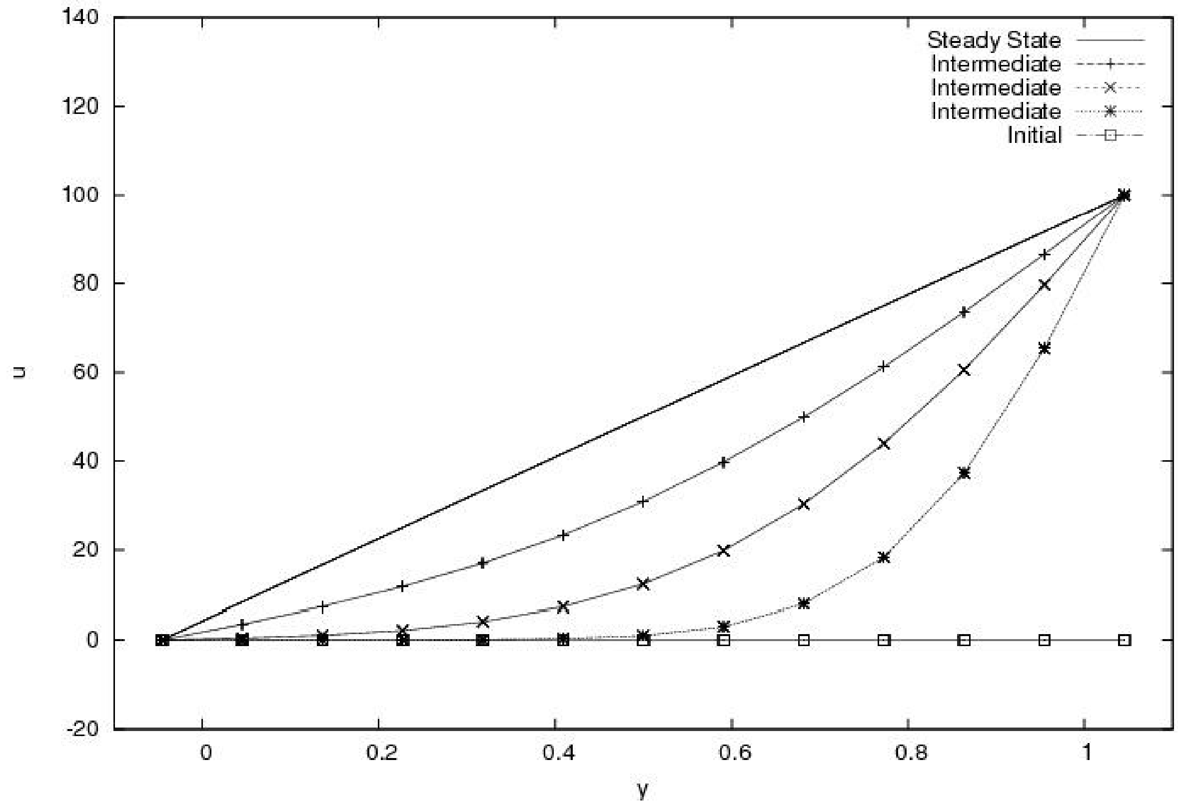
Exact Solution obtained through Oblique Shock relations



3^{rd} order ENO
+
3^{rd} order TVD Runge Kutta
Mach 2.9 Inflow

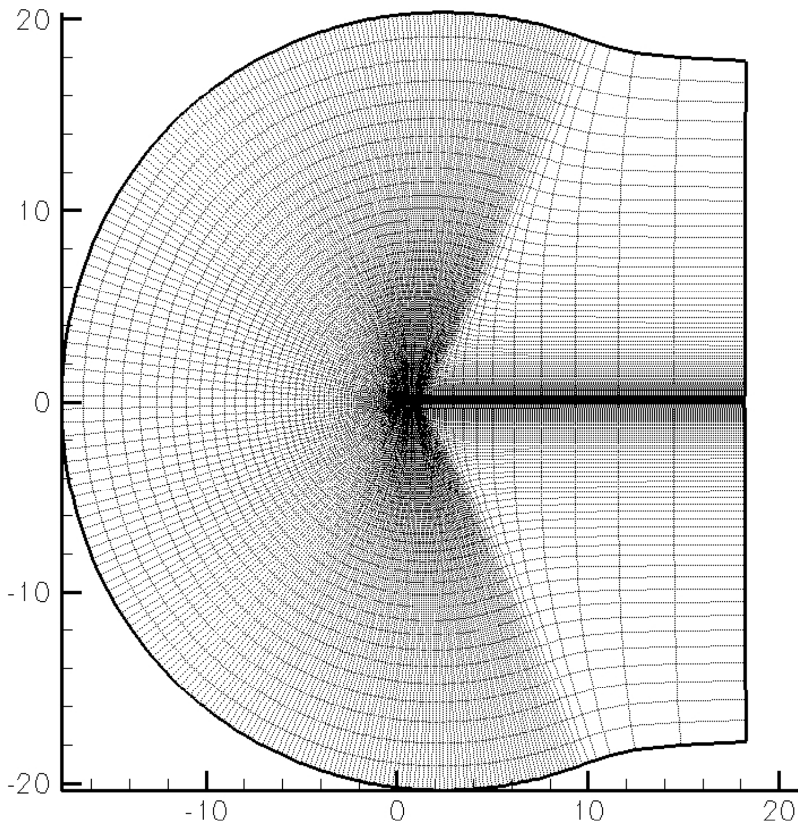
Couette Flow

- Flow between a stationary plate and a moving plate
  (u = 100 m/s)
- Steady state is a linear velocity profile
- 2D Cartesian mesh
- Periodic boundary conditions along x-boundaries
- No-slip conditions along y-boundaries



Initial, intermediate and steady state velocity profiles for Couette flow

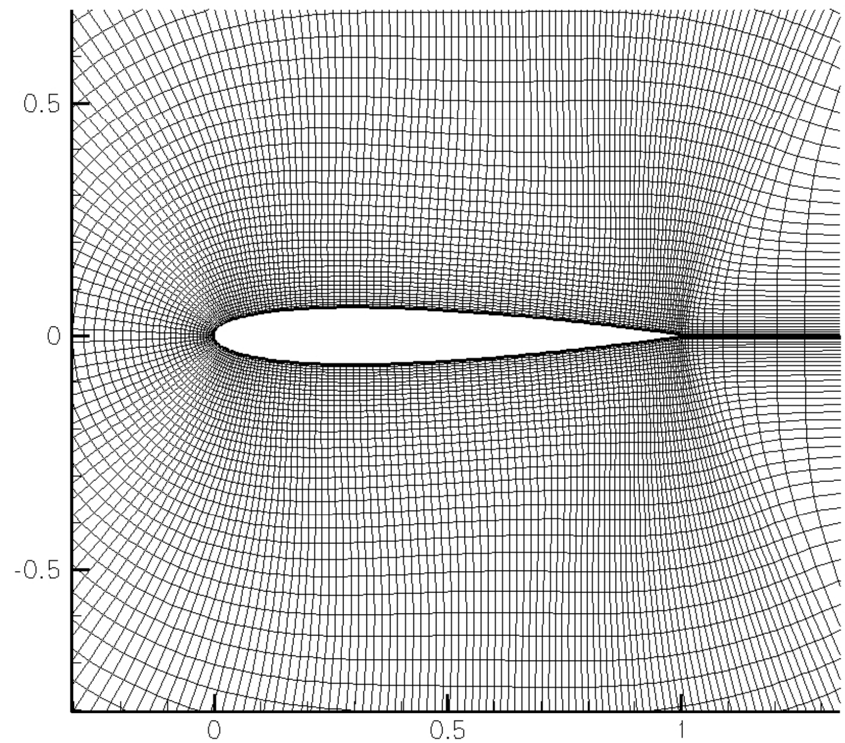
Airfoil Computations – Domain



C-Type Structured Mesh with outside boundary 20 chords away

Freestream boundary conditions on outer boundary

Magnified view of mesh around airfoil (unit chord)

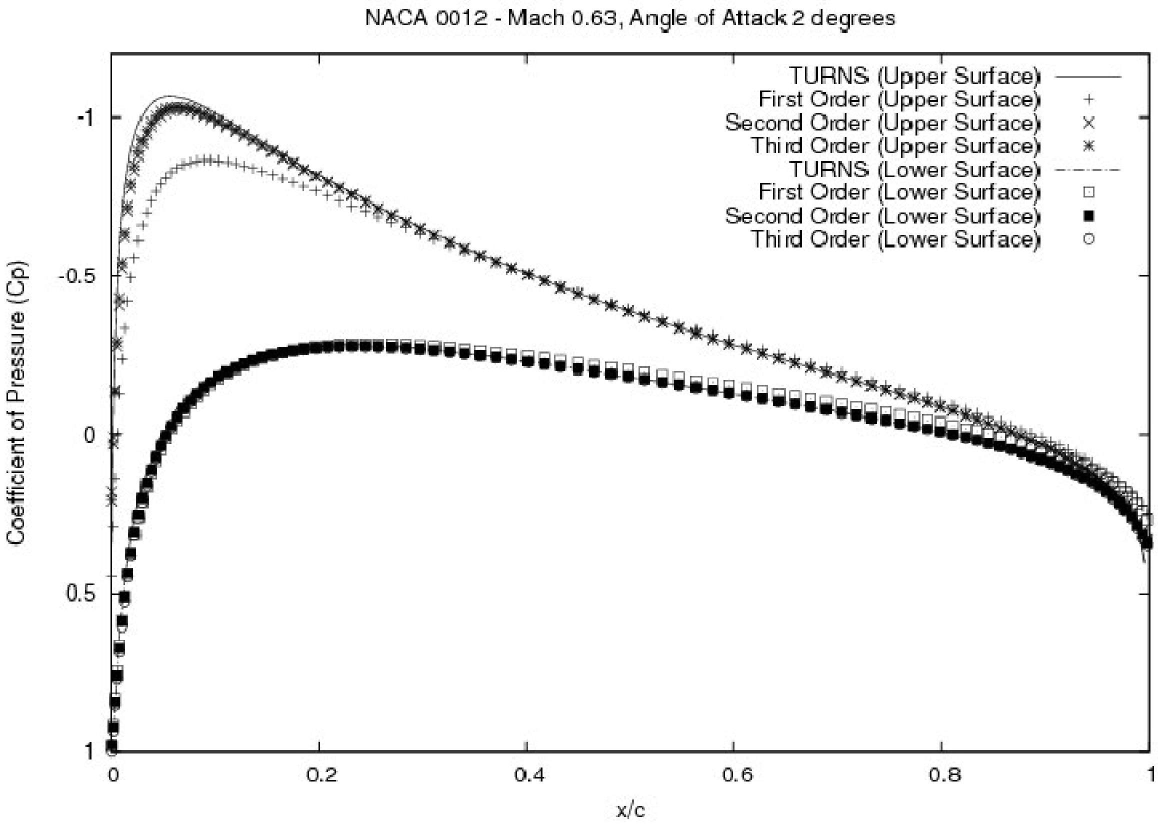


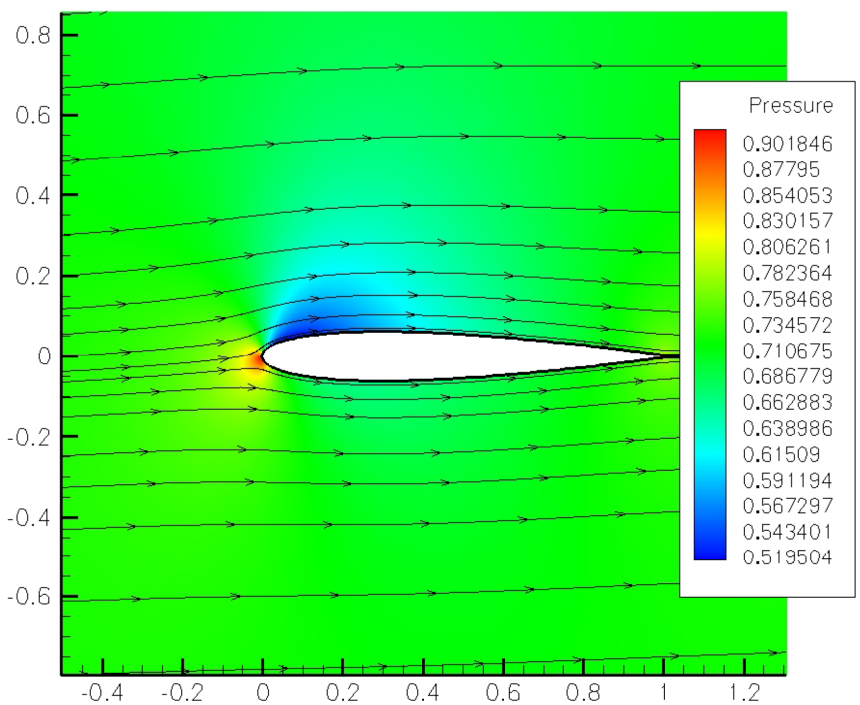
Curved Wall Boundary Conditions at Airfoil Surface

NACA0012 Subsonic (Inviscid)

- Coefficient of Pressure $$C_p = \frac{p - p_\infty}{\rho_\infty u_\infty^2/2}$$

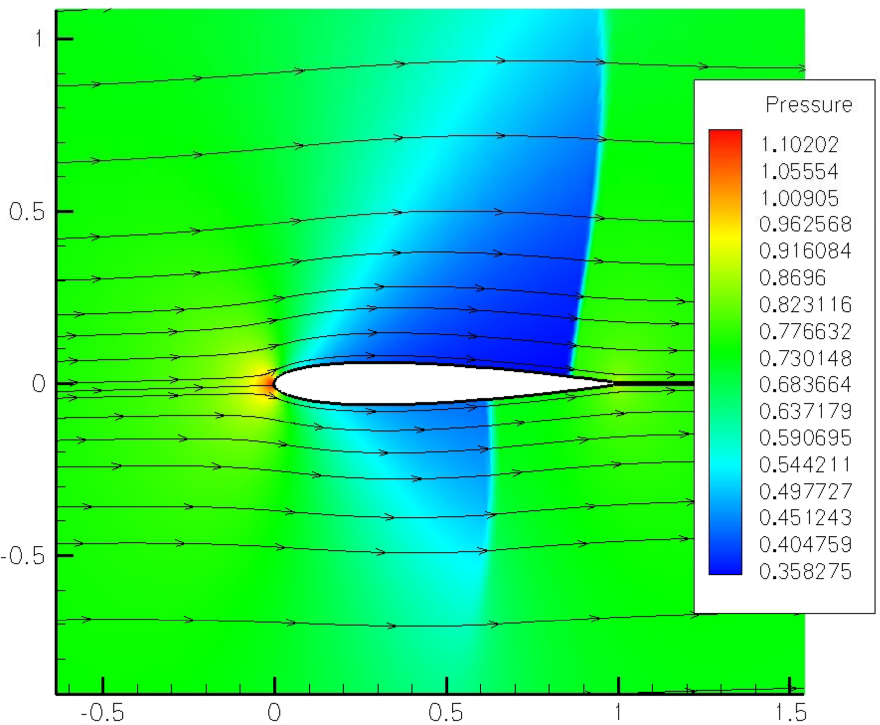
- Results validated with TURNS code





Pressure and streamlines around the airfoil at Mach 0.63 and 2 degrees angle of attack

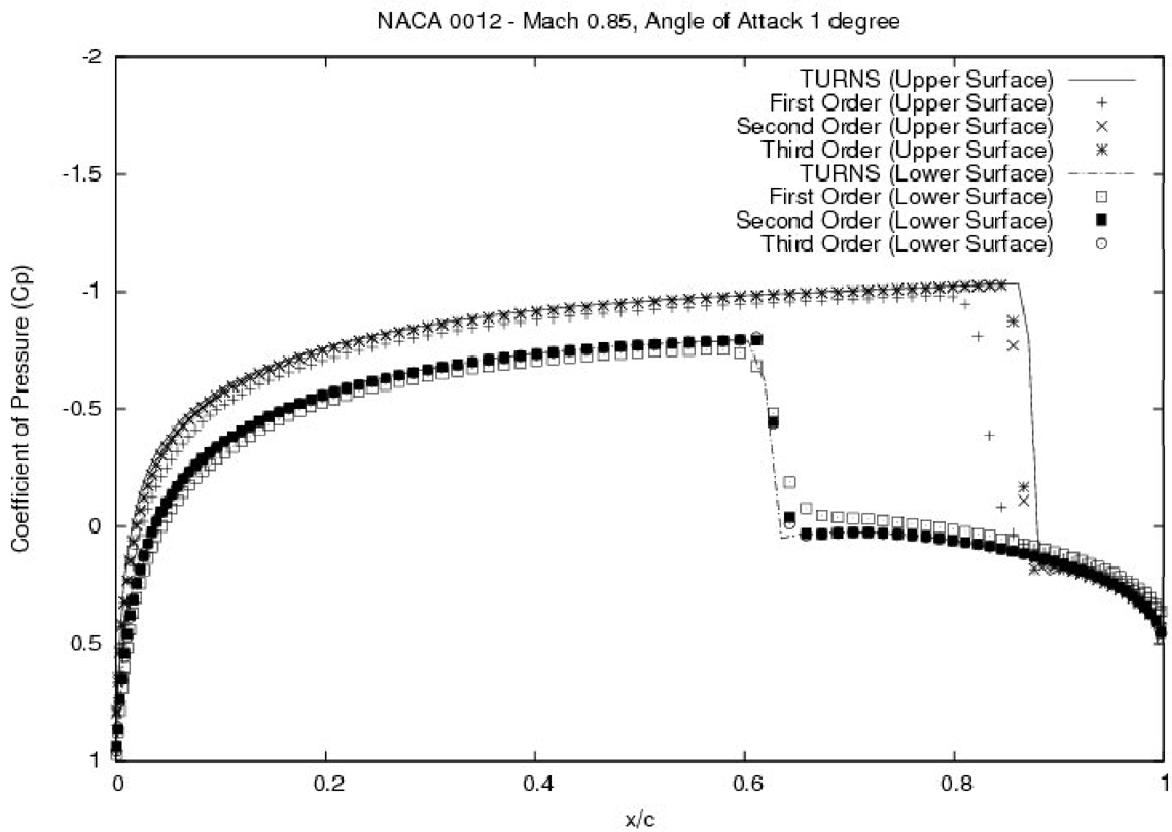
NACA0012 Transonic (Inviscid)



Pressure and streamlines around the airfoil at Mach 0.85 and 1 degrees angle of attack

Results validated with TURNS code

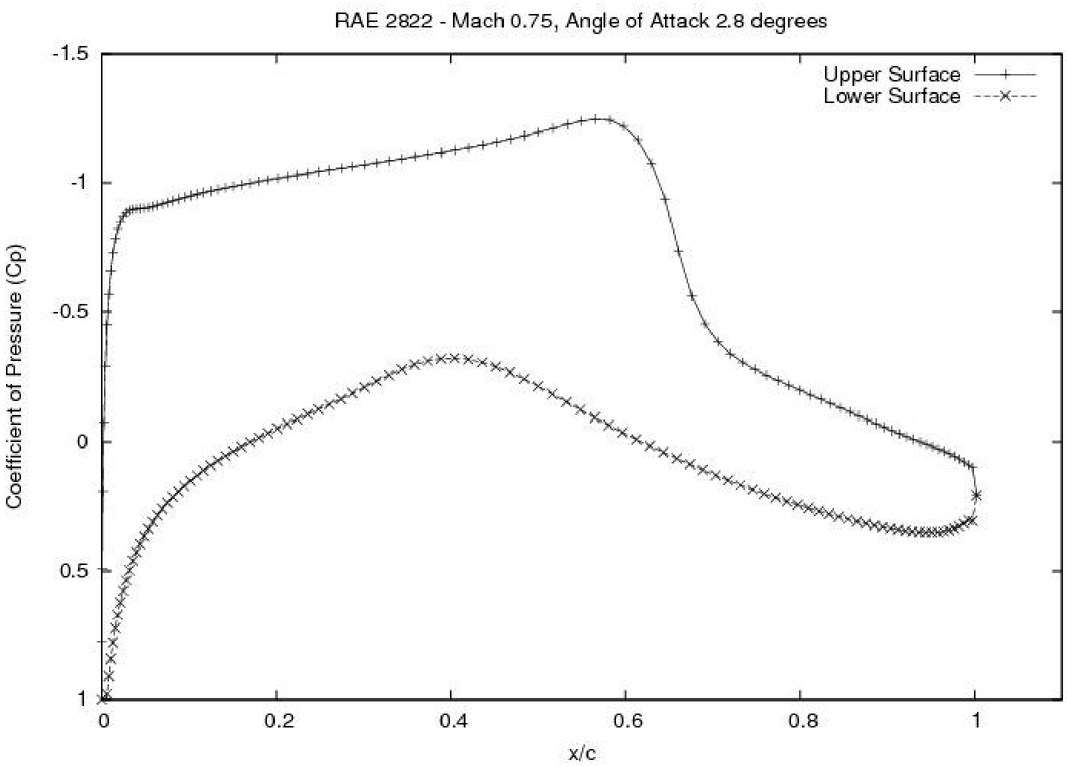
Higher order schemes show better shock resolution than 1st order

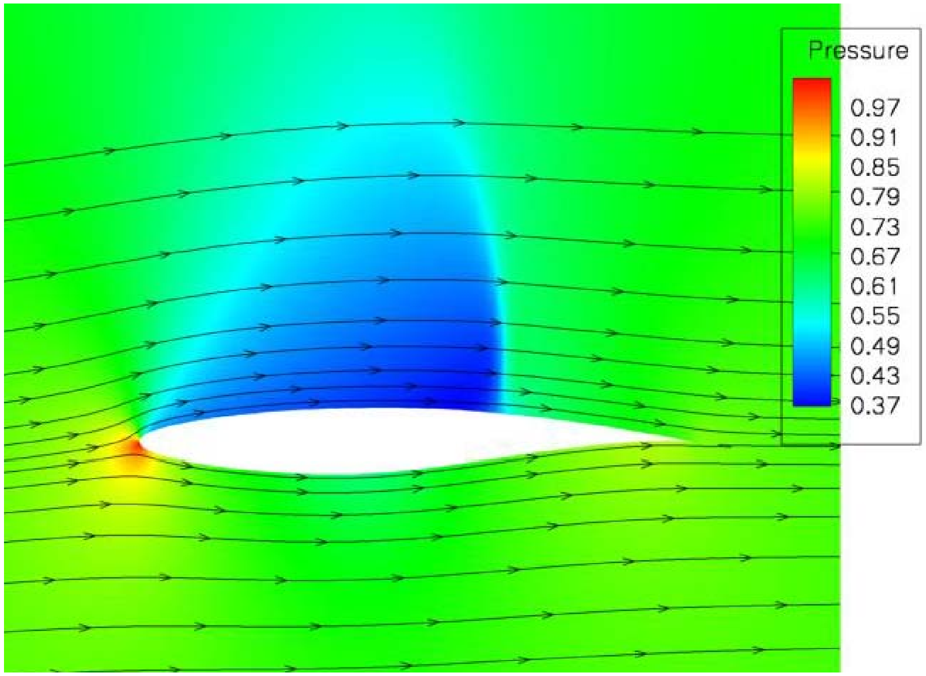


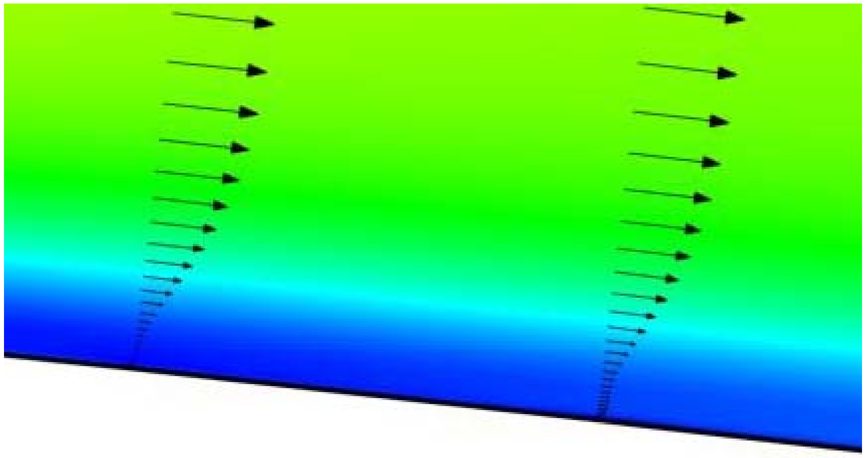
RAE2822 Transonic (Viscous)

RAE 2822 Airfoil at Mach 0.75 and 2.8 degrees angle of attack

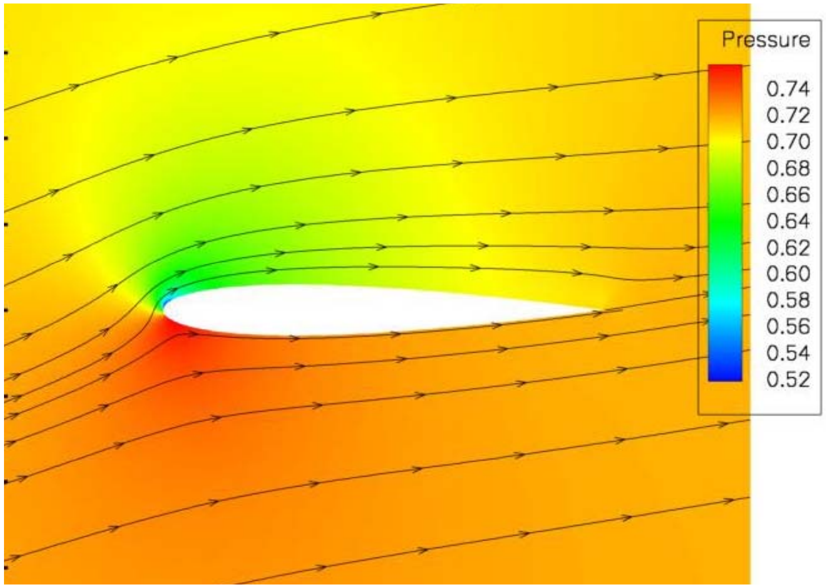
1st Order Roe's scheme with Euler Backward time stepping

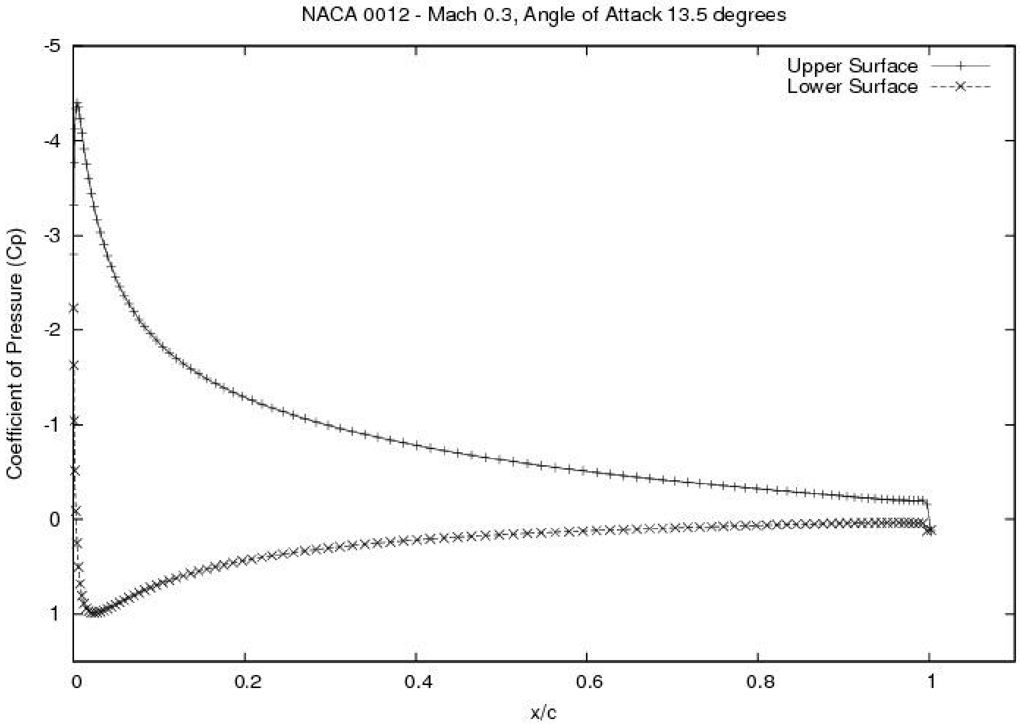


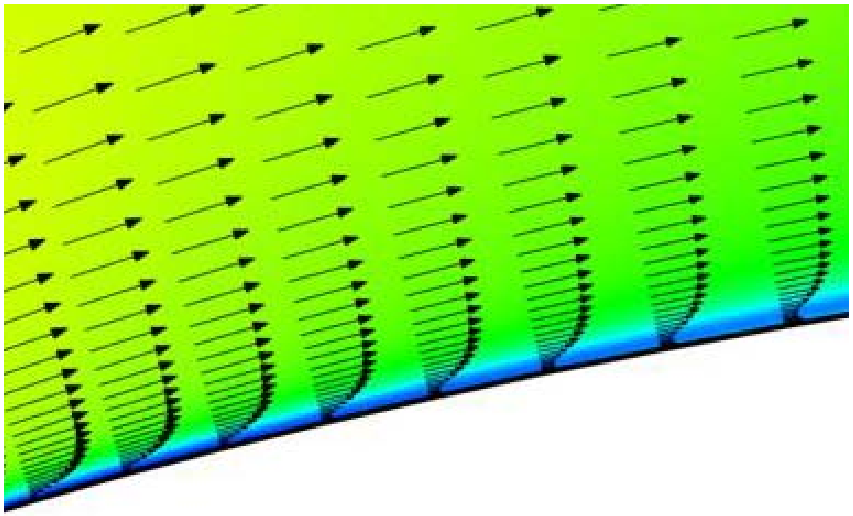




NACA0012 High Angle of Attack (Viscous)







NACA0012 Airfoil at Mach 0.3 and 13.5 degrees angle of attack

1st Order Roe scheme with Euler Backward time stepping

Conclusions

- Flow solver (2D) for inviscid and viscous flow around airfoils
  - Presently validated for steady state computations
  - Post-processing required to compute lift, drag and moment coefficients
  - Efficient solver for banded tri-diagonal system needed
  - Need to incorporate a turbulence model
- Future work – for computation of dynamic stall
  - Increase time accuracy by sub-iterations
  - Validation for unsteady cases (plunging airfoils, rotating airfoils)

End!