A State-Space Model for a Nonlinear Time-Delayed Feedback Loop

Winter Progress Report

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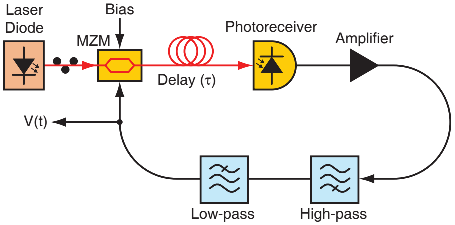
Goal

To implement an alternative, discrete time model for coupled nonlinear (chaotic) time-delayed feedback loops.

System Overview



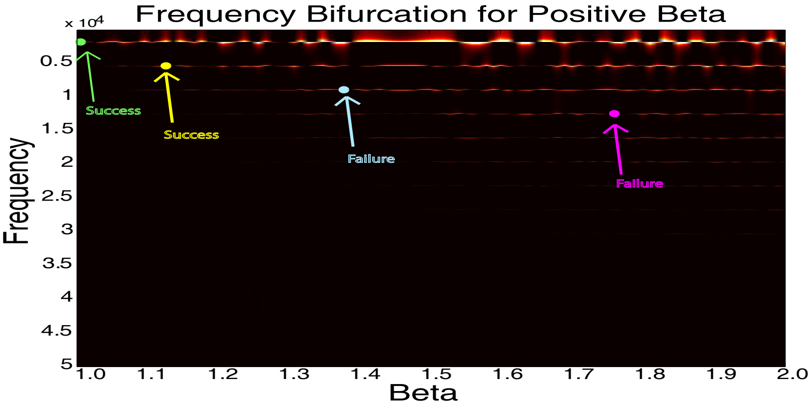
$$\mathbf{u}[n+1] = \mathbf{A}\mathbf{u}[n] + \mathbf{B}\beta cos^2(\mathbf{C}\mathbf{u}[n-k] + \phi)$$

- $u[n]$ is normalized (discrete) RF voltage
- **A,B,C** are matrix coefficients for the band-pass filter
- k is the discrete time delay in the loop
- β is the feedback strength
- ϕ is the phase offset in nonlinearity

Validation Plan

- Single Mach-Zehnder Loop
  - Comparision to Published Analytic Results (Bifurcation points) from Kouomou [1]
  - Comparision to Published Experimental Results of Cohen et. al.[2]
- Coupled Mach-Zehnder Loops
  - Comparision to Open Loop: Argysis [5] (not shown)
  - Comparision to Symmetric 50/50 coupling: Piel [6]

Comparision of Analytic to Simulated Results

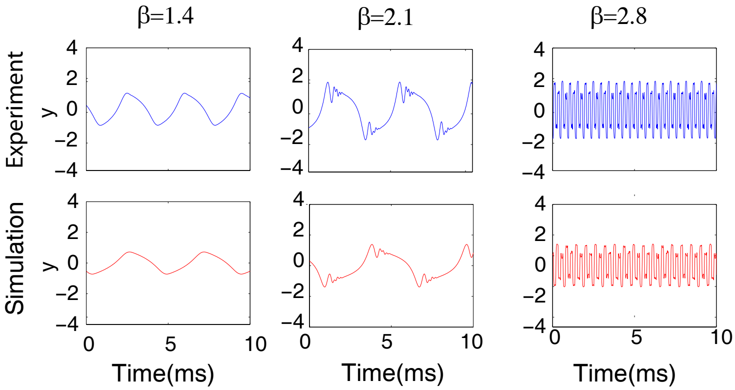


Kouomou predicts the following bifurcations:

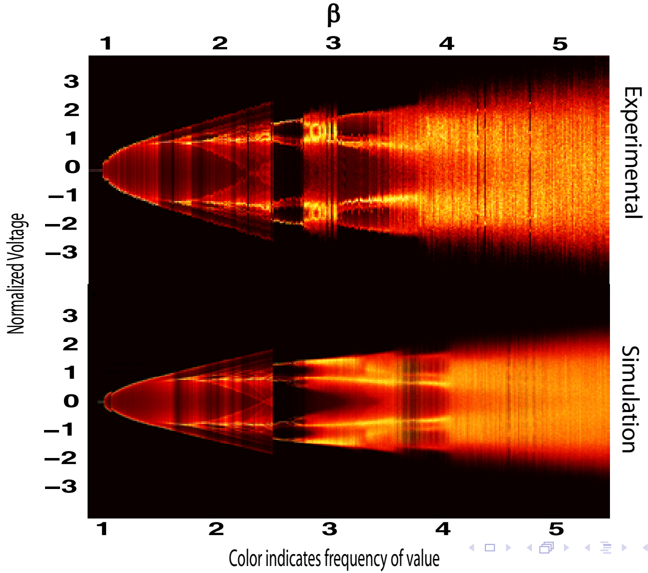
Hopf Bifurcation Points

$\beta_k = (-1)^{k+1}[1 + \frac{(\epsilon R^2 - k^2\pi^2)^2}{2k^2\pi^2 R^2}]$ $\omega_k = k\frac{\pi}{R}$

Comparision of Experimental to Simulated Time Series



Comparision of Experimental to Simulated Bifurcation

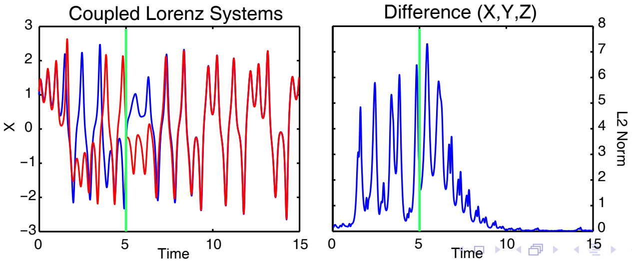


Example of Synchronization in Coupled Lorenz Systems

Coupled Lorenz Equations:

$$\dot{x}_1 = \sigma(y_1 - x_1) + \gamma(x_2 - x_1) \qquad \dot{x}_2 = \sigma(y_2 - x_2) + \gamma(x_1 - x_2)$$
$$\dot{y}_1 = rx_1 - y_1 - 20x_1 z_1 \qquad \dot{y}_2 = rx_2 - y_2 - 20x_2 z_2$$
$$\dot{z}_1 = 5x_1 y_1 - bz_1 \qquad \dot{z}_2 = 5x_2 y_2 - bz_2$$

$$\sigma = 10 \qquad r = 60 \qquad b = \frac{8}{3}$$



Equations for Coupled Mach-Zehnder Loops

We build equations for coupled Mach-Zehnder loops similar to the previous coupled Lorenz System:

(one Mach-Zehnder loop)

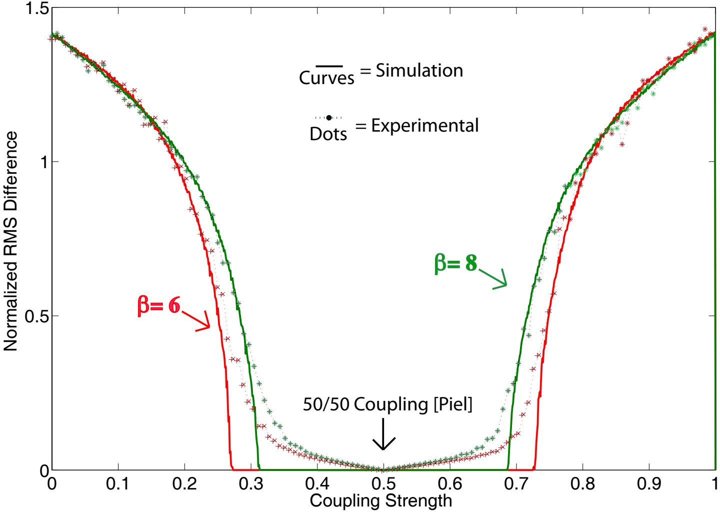
$$\mathbf{u}_1[n+1] = \mathbf{A}\mathbf{u}_1[n] + \ \mathbf{B}cos^2(\mathbf{C}\mathbf{u}_1[n-k] + \phi)$$

(coupled Mach-Zehnder loops)

$$\begin{aligned}\mathbf{u}_1[n+1] = \mathbf{A}\mathbf{u}_1[n] &+ (1-\gamma) * \mathbf{B}cos^2(\mathbf{C}\mathbf{u}_1[n-k] + \phi)\\ &+\gamma \qquad\quad * \ \mathbf{B}cos^2(\mathbf{C}\mathbf{u}_2[n-k] + \phi)\end{aligned}$$

$$\begin{aligned}\mathbf{u}_2[n+1] = \mathbf{A}\mathbf{u}_2[n] &+ (1-\gamma) * \mathbf{B}cos^2(\mathbf{C}\mathbf{u}_2[n-k] + \phi)\\ &+\gamma \qquad\quad * \ \mathbf{B}cos^2(\mathbf{C}\mathbf{u}_1[n-k] + \phi)\end{aligned}$$

Synchronization of Coupled Systems



Milestones

• Implementation and Verification of individual simulations	November 1^{st} (complete)
• Implementation and Verification of final simulation	December 1^{st} (complete)
• Generation of new results	Feburary 1^{st} (in progress)
• Further Expansion of Code	To be Determined

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Dr. Zimin and Dr. Yorke for feedback and improvements

Dr. Roy and Dr. Murphy for initial problem

Adam Cohen and Bhargava Ravoori for experimental results

References

- Kouomou, Yanne. "Nonlinear Dynamics of Semiconductor Laser systems with feedback" *Doctoral Thesis*
- Cohen, Adam, et. al. "Using synchronization for prediction of high-dimensional chaotic dynamics" *Phys. Rev. Let.*, Publication Pending
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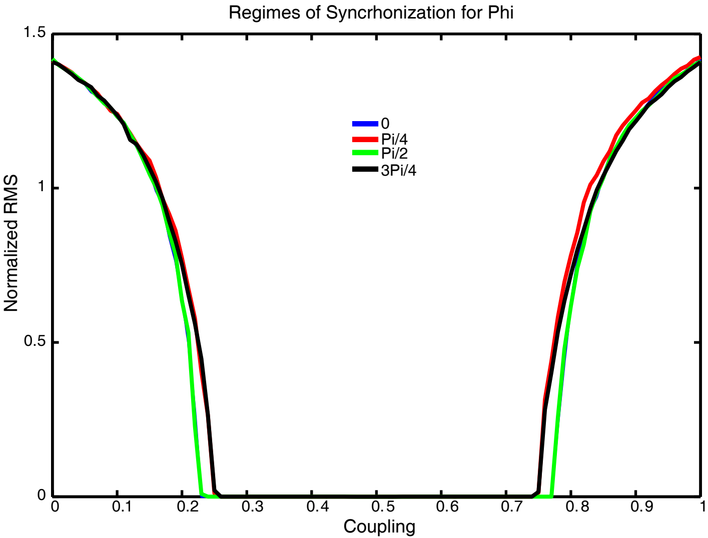
- Piel, Michael, et. al, "Versatile and robust chaos synchronization phenomena imposed by delayed shared feedback coupling" *Phys. Rev. E* Vol 76, 2007 Amer. Phys. Soc.

Use of Code

- Previously Explored
  - 50/50 Coupling (replicated)
  - Synchronization for Coupling vs. Feedback Strength
- To be Explored
  - Synchronization for:
    - Coupling vs. Delay (in progress)
    - Coupling vs. Optical Bias (mostly complete)
  - Noise/Imprecise Parameters (in progress)
  - Quantization
  - Non-Symmetric Coupling
  - Variations in Other parameters

Preliminary Results



Preliminary Results