# IMAGE DEBLURRING - COMPUTATION OF CONFIDENCE INTERVALS

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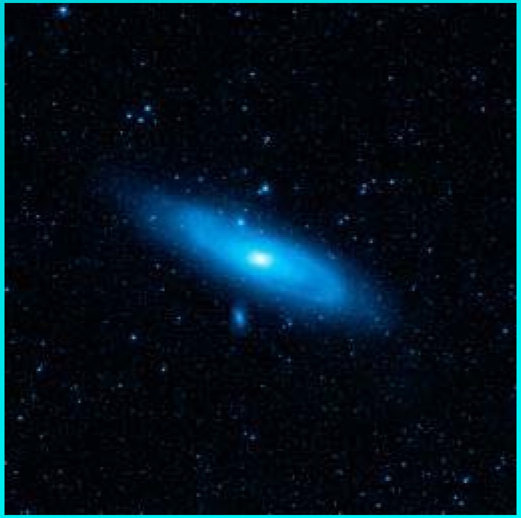
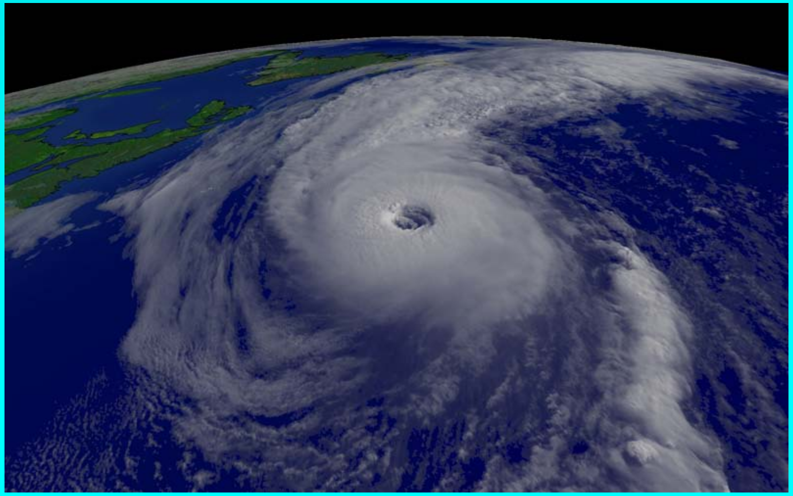
# Introduction



Andromeda nebulae,
picture from NASA



Hurricane Alex,
picture from NOAA

# Blurring

- Machine errors in transforming the image into data
- Background (Light , contrast)
- Motion of the object, motion of the camera

# Example 1

**Clear Image**

**Blurred Image**

# Example 2

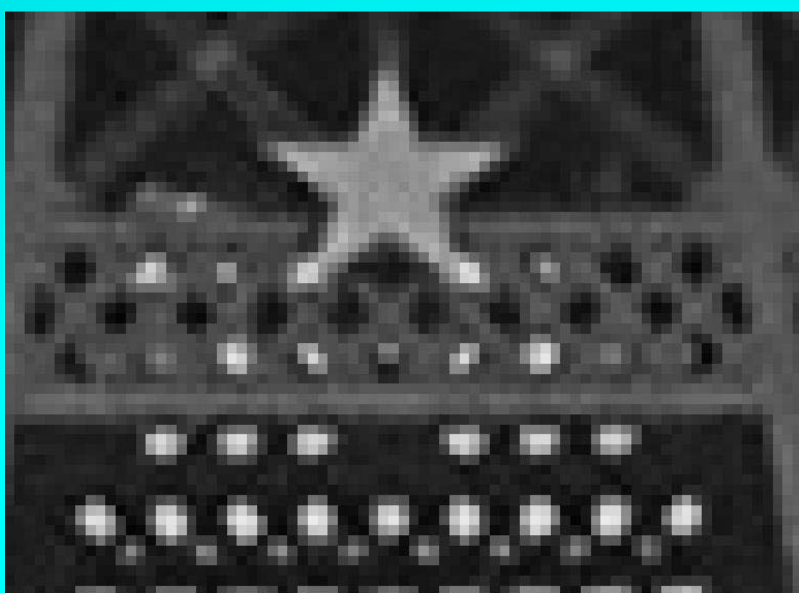
**Clear Image**

**Blurred Image**

# Pixels

Pixel Values [0,255]

0: black
255: white



# Notation

| Symbol | Size | Explanation |
|---|---|---|
| **K** | mxn | Matrix defined through the Point Spread function (PSF) in the case of a linear problem |
| X | | Original Clear Image |
| **x** | nx1 | Vector containing the values corresponding to the pixels of the image X |
| B | | The blurred image we measure |
| **b** | mx1 | Vector which contains the values of the pixels of the blurred image B |
| **e** | mx1 | Vector of noise |

# Model

In the case of linear Point Spread Function

$$\mathbf{b} = \mathbf{K}\mathbf{x} + \mathbf{e}$$

# Problem

- Given $\mathbf{b}$, matrix $\mathbf{K}$ and a distribution for the noise $\mathbf{e}$ with mean value 0 and variance a matrix $\mathbf{S}^2$, we need to compute confidence intervals for the quantities $\varphi_k^* = \mathbf{w}_k^T \cdot \mathbf{x}$, for $k = 1,2,\ldots,p$ where $\mathbf{w}_k$ are given vectors.

# Confidence intervals

- Intervals into which the values of a parameter fall, with a certain probability.

- Example: $100\alpha\%$ confidence interval, $\alpha \leq 1$

  One at a time confidence intervals

$$\Pr\{l_k \leq \varphi_k^* \leq u_k\} = \alpha, k = 1,2,\ldots, p$$

  Simultaneous confidence intervals

$$\Pr\{l_k \leq \varphi_k^* \leq u_k , k = 1,2,\ldots,p\} \geq \alpha$$

- Suppose that the noise is normally distributed, and that $\hat{\mathbf{x}}$ is an unbiased estimate of the true solution . Then, given $\alpha$ in (0,1), there is a $100\alpha\%$ probability that the true value of $\mathbf{w}^T\mathbf{x}$ is contained in the interval $[l,u]$ where $l = \min_{\mathbf{x}} \left\{ \mathbf{w}^T\mathbf{x} : \left\| \mathbf{K}(\mathbf{x}-\hat{\mathbf{x}}) \right\|_{\mathbf{S}}^2 = \kappa^2 \right\}$
  and , $u = \max_{\mathbf{x}} \left\{ \mathbf{w}^T\mathbf{x} : \left\| \mathbf{K}(\mathbf{x}-\hat{\mathbf{x}}) \right\|_{\mathbf{S}}^2 = \kappa^2 \right\}$

  Where $\left\| \mathbf{K}(\mathbf{x}-\hat{\mathbf{x}}) \right\|_{\mathbf{S}}^2 = \left[ \mathbf{K}(\mathbf{x}-\hat{\mathbf{x}}) \right]^{\mathrm{T}} \mathbf{S}^{-2} \left[ \mathbf{K}(\mathbf{x}-\hat{\mathbf{x}}) \right]$ and $\kappa$ is such that for the probability $\alpha$ , i.e., for confidence $100\alpha\%$
  $\alpha = \int_{-\kappa}^{\kappa} n(\mathbf{x};0,1)\,d\mathbf{x}$ where $n(\mathbf{x};0,1)$ is the normal

  distribution of $\mathbf{x}$ with mean value 0 and variance 1.

- With the same assumptions , $\mathbf{x} \geq 0$ and $\mathbf{S}$ nonsingular and symmetric, the probability that $\varphi^*$ is contained in the interval $[l, u]$ is greater than or equal to $\alpha$ where

$$l = \min_{\mathbf{x}} \left\{ \mathbf{w}^T \mathbf{x} : \left\| \mathbf{K}(\mathbf{x} - \hat{\mathbf{x}}) \right\|_{\mathrm{S}} \leq \mu, \mathbf{x} \geq 0 \right\}$$

and $$u = \max_{\mathbf{x}} \left\{ \mathbf{w}^T \mathbf{x} : \left\| \mathbf{K}(\mathbf{x} - \hat{\mathbf{x}}) \right\|_{\mathrm{S}} \leq \mu, \mathbf{x} \geq 0 \right\}$$

$\operatorname{rank}(\mathbf{K}) = q$ , $\alpha = \int_0^{\gamma^2} \chi_q^2(\rho) d\rho$ , $r_0 = \min_{\mathbf{x}} \left\| \mathbf{K}(\mathbf{x} - \hat{\mathbf{x}}) \right\|_{\mathrm{S}}^2$ , $\mu^2 = r_0 + \gamma^2$

and $\chi_q^2$ is the probability density function for the chi-squared distribution with $q$ degrees of freedom.

# The matrix **K**

- The matrix **K** is defined through the Point Spread function (PSF) and can be determined by using a single point source in a pixel of an image and then moving it to other pixels of the image to obtain more blurred images. Knowing the clear image and the blurred image for all the pixels we can compute the matrix **K**.
- In the following, the matrix **K** will be a given nxn square matrix.

# Sub-matrices

- We want to reduce the size of the matrix (and so of the image) to facilitate the computations. Suppose that $\mathbf{b} = \mathbf{K}\mathbf{x}$ . For a sub-image rxc we proceed as follows: We have two matrices $\mathbf{E}$ and $\overline{\mathbf{E}}$ where $\mathbf{E}$ has rc columns of unit vectors and $\overline{\mathbf{E}}$ has n-rc columns of unit vectors. Then,

$$\mathbf{E}^T\mathbf{K}\mathbf{x} = \mathbf{E}^T\left(\mathbf{K}[\mathbf{E}\overline{\mathbf{E}}]\right)\left(\begin{bmatrix}\mathbf{E}^T \\ \overline{\mathbf{E}}^T\end{bmatrix}\mathbf{x}\right) = \begin{pmatrix}\mathbf{K}_s & \mathbf{K}_t\end{pmatrix}\begin{pmatrix}\mathbf{x}_s \\ \mathbf{x}_t\end{pmatrix} = \mathbf{K}_s\mathbf{x}_s + \mathbf{K}_t\mathbf{x}_t$$

  where $\mathbf{x}_s$ is the vector corresponding to the sub-image of the original image. Ignoring the second term of the right hand side, we obtain $\mathbf{b}_s = \mathbf{K}_s\mathbf{x}_s$

# Implementation

- Matlab (IPT- Image Processing toolbox)

- Case of spatially invariant blur
  Smaller Problems- same matrix
  Parallel Computing (Open MP or MPI)

# Databases

- http://sipi.usc.edu/database/

USC-SIPI Image Database

Signal & Image Processing

Institute of the University

of Southern California

- http://www.imageprocessingplace.com/root_files_V3/image_databases.htm

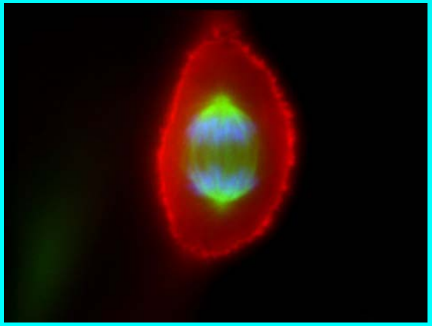


Image Processing Place

# Validation

- Known data $\longrightarrow$ Expected results

- Take images
- Blur images
- Add noise
- Use code to compute the confidence intervals for these images
- Deblur images
- Count samples in the computed intervals

- If close to 95% fall in the 95% confidence intervals, then the code is validated.
- If not… check code, correspondence with theory and special circumstances

# Testing

**Good code: Right results in relatively short time**

Depends on

- Size of image
- Format of image
- Various Point Spread Functions

# Schedule-Milestones

- September: literature, image processing toolbox of Matlab, Project Proposal
- October: literature, problem, code
- November: code, validation
- Early December: midyear report
- Late January: testing
- February: parallel computing
- March: testing, validation
- April: validation, final report
- May: final presentation

# REFERENCES


[1] Tony F. Chan and Jianhong (Jackie) Shen, “Image Processing and Analysis”, SIAM, Philadelphia, 2005

[2] Per Christian Hansen, James G. Nagy and Dianne P. O’Leary, “Deblurring Images Matrices, Spectra, and Filtering”, SIAM, Philadelphia, 2006

[3] James G. Nagy and Dianne P. O’Leary, “Image Restoration through Subimages and Confidence Images”, Electronic Transactions on Numerical Analysis, 13, 2002, p 22-37

[4] Dianne P. O’Leary and Bert W. Rust, “Confidence Intervals for inequality constrained least squares problems, with applications to ill-posed problems”, SIAM Journal on Scientific and Statistical Computing, 7, 1986, p 473-489

[5] Bert W. Rust and Dianne P. O’Leary, “Confidence intervals for discrete approximations to ill-posed problems”, The Journal of Computational and Graphical Statistics, 3, 1994, p 67-96

Thank you