Automated Parameter Selection Tool for Solution to Ill-Posed Problems: *An Application to Image Processing*

Midyear presentation

Brianna R. Cash[1]   Advisor: Dianne O'Leary[2]

[1]Applied Math Scientific Computing (AMSC)
University of Maryland, College Park

[2]Department of Computer Science
University of Maryland, College Park

AMSC 663, 2011

Outline

1. Motivation
   - An application to medical images
2. Project Goals
   - Outline of project
3. Update on Implementation
   - Total variation based regularization method
   - Diagnostics
   - Frontend
4. Schedule and Milestones
   - Completed milestones
   - Remaining milestones

Outline

The problem.

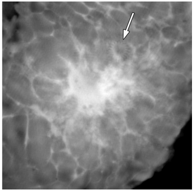

Figure: Stevens G M et al. Radiology 2003;228:569-575

- Medical images can be expensive to produce.
- Used for making medical decisions
- Images can be distorted and/or noisy.
  - physics of the measurement
  - non-homogenous material (humans)

Deblurring/Denoising medical images:
The discrete model: $\mathbf{Ax} + \epsilon = \mathbf{b}$, $\epsilon \sim \mathbf{N}(\mathbf{0}, \mathbf{S^2})$ where

- $\mathbf{A}$ is a known $m \times n$ matrix where $m \geq n$ (Blurring matrix)
- $\mathbf{x}$ is unknown $n \times 1$ vector (true image)
- ϵ is a $m \times 1$ vector (noise)
- $\mathbf{S^2}$ is known $m \times m$ (variance matrix for ϵ)
- $\mathbf{b}$ is a known $m \times 1$ vector (blurred and noisy image)

Inherent to image deblurring problems, $\mathbf{A}$ is ill-conditioned:

- Solve by replacing the problem with an approximate well-posed problem by introducing a constraint or regularization parameter

Formulation of regularization problem:

$$\min \|\mathbf{Ax} - \mathbf{b}\|_2^2 + \gamma\Omega(x)$$

- Where $\Omega(x)$ is smoothing function or penalty function and γ is the regularization parameter.

Tikhonov: $\Omega(x) = \|\mathbf{x}\|_2^2$.
Total Variation: $\Omega(x) = TV(\mathbf{x})$ where $TV(\mathbf{x}) = \|\nabla\mathbf{x}\|_1$.

Selecting a good regularization parameter:
Expensive, problem dependent, and subject to bias:

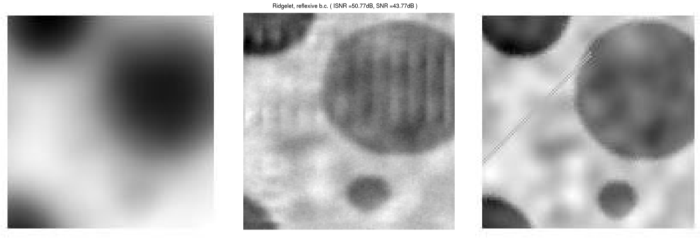

Figure: Images courtesy of Dianne O'Leary

But what is unexpected is often what we are interested in!

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Project Goal: Building a software package for parameter selection

Frontend

- Graphical User Interface (GUI) built using Matlab's GUI toolbox

Backend

- Regularization method
  - Regularization methods from *RestoreTool* [Nagy2002]
  - Implement code for Total Variation regularization method
- Method for initial parameter selection
  - Generalized Cross-Validation (GCV) in *RestoreTool* for regularization methods included.
  - Implement code for GCV for Total Variation
- Validate candidate solutions using statistical diagnostics
  - Adapt existing code for statistical diagnostics from Dianne O'Leary

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Total variation based regularization method:

$$\min \|\mathbf{Ax} - \mathbf{b}\|_2^2 + \gamma TV(\mathbf{x}) \text{ where } TV(\mathbf{x}) = \|\nabla \mathbf{x}\|_1.$$

Newton Method with Conjugate Gradient (CG)[Chan1996]-

- Discrete formulation of TV term: $\min \frac{1}{2}\|\mathbf{Ax} - \mathbf{b}\|_2^2 + \lambda \sum_{i=1}^{n} \sqrt{\|\mathbf{D}_i^T \mathbf{x}\| + \beta}$ where $\mathbf{D}_i^T \mathbf{x} = (x_{i+1} - x_i, x_{i+n_h} - x_i)$ with Neumann boundary conditions.
- The first order condition is: $\mathbf{g}(\mathbf{x}) = 0$ and the Hessian: $\mathbf{H}(\mathbf{x})$

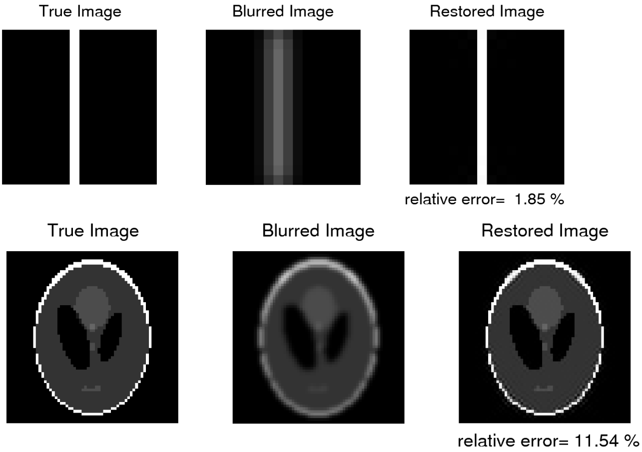
Newton-CG:

1. Initialize: $\mathbf{x}_0$ and initial approximation $\mathbf{x}_*$, $k = 0$
2. If $\mathbf{x}_k$ is “good enough”, terminate
3. Solve $\mathbf{H}(\mathbf{x})\mathbf{p_k} = -\mathbf{g}(\mathbf{x})$ where $\mathbf{p}$ is the Newton direction using conjugate gradient (CG)
4. Set $\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{p}_k$ where α_k is determined by a linesearch.
5. Set $k = k + 1$

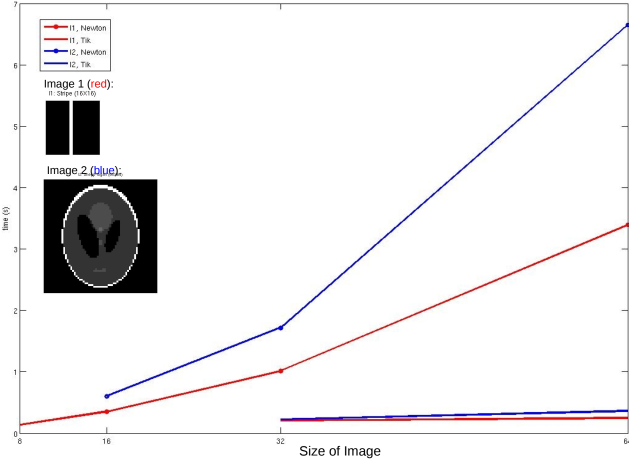
Strive for Low Storage Implementation

- Use CG so that the Hessian does not need to be stored explicitly
  $\mathbf{Hv} = \mathbf{H}(\mathbf{x})\mathbf{v}$ where $\mathbf{v}$ is an arbitrary vector.
- The blurring matrix is not stored explicitly.
  - Use sparse representation for small images
  - Store the Point Spread Function (PSF) and the boundary condtions type (where the $m \times n$ blurring matrix is formed from the PSF and boundary condtions) for larger images.

Initial testing of Newton Method-CG



Timing for Low storage Newton vs. Tikhonov regularization (RestoreTool)



Validation of Newton Method-CG (ongoing)

- Programed modularly so that each piece (Newton Step, CG, function evaluations) can be validated
  - CG was validated for small $\mathbf{Ax} = \mathbf{b}$ test problems where the results could be verified.
  - Implementation of the function for the minimization function $f(\mathbf{x})$, gradient $\mathbf{g}(\mathbf{x})$, Hessian times a vector $\mathbf{Hv}(\mathbf{x}, \mathbf{v})$ were verified.
- Direct implementation of the Newton Method (without CG) was also implemented and was compared to the results to the low-storage Newton method with CG and linesearch.
- Used binary test images without noise and verified the results were close to the true image.

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Motivation for Statistical Based Diagnostics.

$$\mathbf{b} = \mathbf{A}\mathbf{x} + \epsilon$$

Assumptions: The noise ϵ where $\epsilon \sim N(\mathbf{0}, \mathbf{I_m})$
$\mathbf{x}^*$ is the estimate of $\mathbf{x}$ the the residual vector is

$$\mathbf{r} = \mathbf{b} - \mathbf{A}\mathbf{x}^*$$

Where we expect $\mathbf{r} \sim \mathbf{N}(\mathbf{0}, \mathbf{I_m})$
(If $\epsilon \sim N(\mathbf{0}, \mathbf{S^2})$ the problem can be rescaled by the matrix $\mathbf{S^{-1}}$)

This characteristic of the residual inspired three diagnostics [RustOleary2008]

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Diagnostics Implementation

- *Diagnostic 1*: $\|\tilde{\mathbf{r}}\|_2^2 \in [m - 2\sqrt{2m}, m + 2\sqrt{2m}]$ where $m = E[\|\epsilon\|_2^2]$.
- *Diagnostic 2*: Goodness of fit of the normal curve to the histogram using Matlabs function *chi2gof*.
    - The implementation of this diagnostic may be replaced by the Fisher Normality Test [RustOleary2008] based on *checkkperiod.m*.
- *Diagnostic 3*: Cumulative periodogram of the residual is within 95% confidence band of the cumulative periodogram of the time series of white noise [RustOleary2008], based on *checkkperiod.m* by Dianne 0'Leary.

For 1000 runs, number of times the Diagnostics are NOT satisfied:

Residual	Diag. 1	Diag. 2	Diag. 3	Fisher
$r_n \sim N(0,1)$	51	46	14	48
$r_n + I(i)$	950	999	539	1000
$r_n + .05 * r_p \; r_p \sim pois(1)$	156	1000	149	1000

$I(i) = 1$ if $(i-1) mod(100) = 0$ or $(i-2) mod(100) = 0$ and $I(i) = 0$ otherwise.

Diagnostics Testing

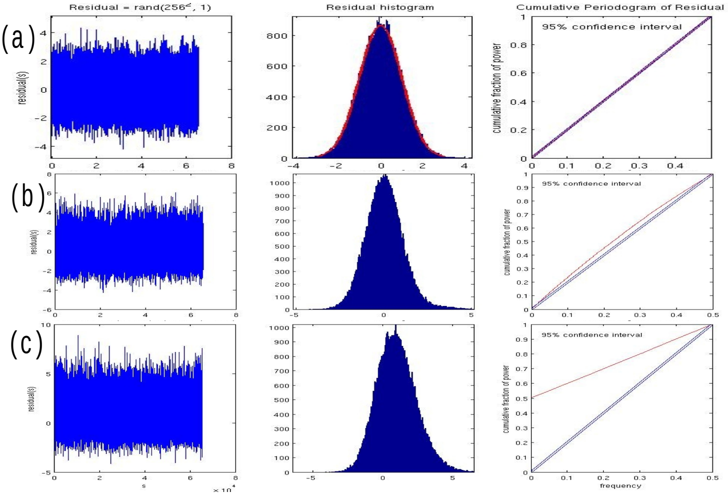

Figure: Plots of residual, histogram of residual, cumulative periodogram
(a) Normal distributed, (b) Normal plus Poisson (1), (c) Normal plus features

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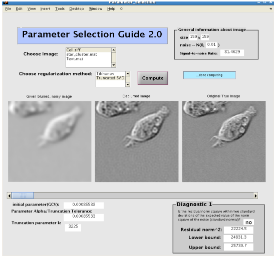
Parameter Selection 2.0 GUI



Brief Demonstration

Parameter Selection 2.0 GUI Remaining Challenges:

- Issue with setting min and max value of slider
- Portability between systems

Outline

Completed- Semester 1 Milestones:

- Learn about and become comfortable with RestoreTool (object oriented programming) and Matlab GUI.
- **Oct 15 - Milestone**: A basic frontend for RestoreTool regularization method.
- **Nov 30 - Milestone**: Validated statistics based diagnostics in RestoreTool framework.
- Read literature on Total Variation regularization and determine iterative method.
- **Dec 1 - Milestone**: Develop a GUI for parameter selection using the diagnostic test and RestoreTool.
- **Dec 15 - Milestone**: Outline of Total Variation regularization method and basic implementation Matlab.
- **Dec 15 - Milestone**:Deliver mid-year report.

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Second Semester Milestones

- Read literature about High-Performance computing and Matlab's parallel toolbox.
- **Feb 1 - Milestone**: Total Variation regularization in RestoreTool framework.
- **Feb 15 - Milestone**: Validation of Total Variation regularization tool.
- **Feb 30 - Milestone**: Generalized Cross Validation for Total Variation tool.
- **Mar 15 - Milestone**: Validation of Generalized Cross Validation for Total Variation tool.
- **Mar 30 - Milestone**: Develop GUI for parameter selection to include regularization method from RestoreTool along with the Total Variation tool.
- **April 15 - Milestone**: Optimize software package (using parallel toolbox in Matlab if available).
- **April 30 - Milestone**: Deliver final software package.
- **May 15 - Milestone**: Deliver Final presentation.

References I

- Tony F. Chan and Jianhong Shen.
  Image Processing and Analysis.
  SIAM, Philadelphia, PA, 2005.
- Wayne A. Fuller.
  Introduction to Statistical Time Series.
  SIAM, Philadelphia, PA, 1998.
- Per Christian Hansen.
  Rank-Deficient and Discrete Ill-Posed Problems:Numerical Aspects of Linear Inversion.
  Wiley-Interscience, New York, NY, 1996.
- Per Christian Hansen.
  Regularization Toolbox.
  http://www2.imm.dtu.dk/ pch/Regutools/regutools.html.

References II

- C.T. Kelly.
  Iterative Methods for Linear and Nonlinear Equations.
  SIAM, Philadelphia, PA, 1995.
- Stephen G. Nash.
  Linear and Nonlinear Programming.
  McGraw-Hill, 1996.
- James G. Nagy.
  RestoreTool.
  http://www.mathcs.emory.edu/ nagy/RestoreTools/.
- Dianne P. 0'Leary.
  Scientific Computing with Case Studies.
  SIAM, Philadelphia, PA, 2009.

References III

- Tony F. Chan, Gene H. Golub and Pep Mulet.
  A nonlinear primal-dual method for total variation-based image restoration.
  Lecture Notes in Control and Information Sciences, Vol.219, pp.241-251, 1996.
- Katrina P. Lee, James G. Nagy and Lisa Perron.
  Iterative Methods for Image REstoration: A Matlab Object Oriented Approach
  Numerical Algorithms, Vol.36, pp.73-93, 2004.
- James G. Nagy, K. Palmer and L. Perrone.
  Iterative Methods for Image Deblurring: A Matlab Object Oriented Approach.
  Numerical Algorithms. 36: 73-93, 2004.

References IV

Bert W. Rust and Dianne P. O'Leary.
Residual programs for choosing regularization parameters for ill-posed problems.
Inverse Problems. 24:034005 (30 pages), 2008. (invited papaer

Bert W. Rust.
Parameter selection for constrained solutions to ill-posed problems.
SIAM Journal on Scientific Computing, Vol.17, pp.227-238, 1996.