FINAL REPORT: TOPOGRAPHY IN LARGE-EDDY SIMULATION

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PROBLEM FORMULATION

How can we include topography into large-eddy simulation, through the immerse boundary method, to assist the state-of-the-art solver, Oceananigans.jl, in answering the many research questions involving flow along and over topography?

Ramadhan, et al. (2020)

OUTLINE

Background

Motivation

Methodology

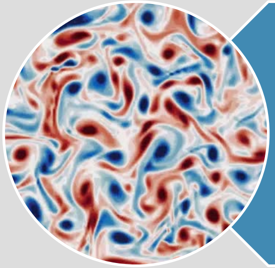
Implementation

Results

BACKGROUND:

Climate Modeling Alliance (CliMA) aims to build climate model from scratch

As part of model, Oceananigans.jl solves incompressible fluid problems of ocean



Oceananigans.jl uses Large Eddy Simulations (LES) because they can resolve the turbulent motions of ocean domains at the necessary scales

MOTIVATION: IMPORTANCE OF TOPOGRAPHY IN OCEAN SIMULATIONS

Gula, (2016), McWilliams, (2016), Wenegrat, (2020),(2018)

Dissipation of Energy

- Topography can extract energy from geostrophic flows (10-100 km), creating sub-mesoscale turbulence (0.1 – 10 km)

Nutrient Mixing

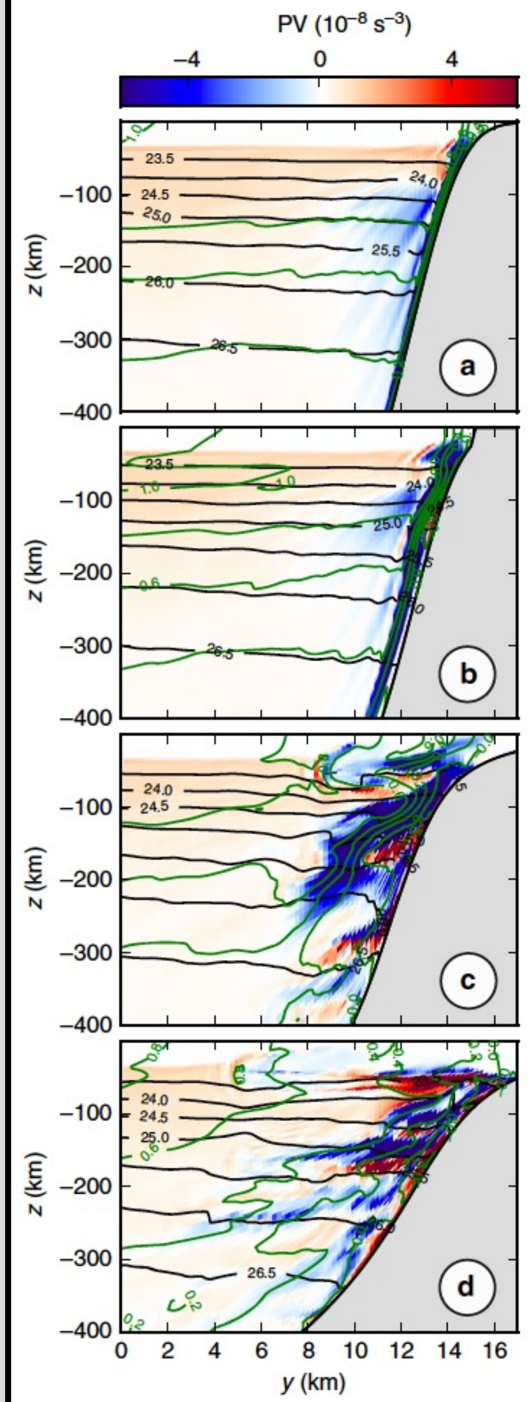
- Complex bottom boundary layer can cause flow to separate from wall and mix with interior

Differences from Surface

- Unknown differences between sea surface layer and bottom boundary layer (BBL)

Limited Topography options in LES

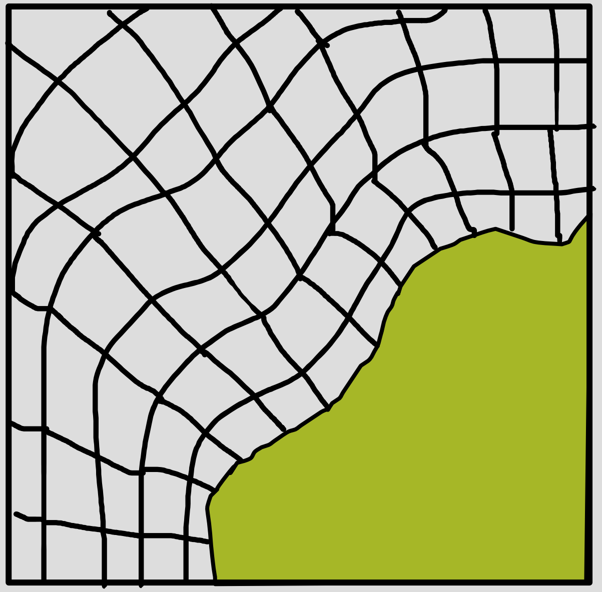
- No wide-spread, open-source LES options that include topography



METHODOLOGY: IMMERSED BOUNDARY METHOD (IBM)

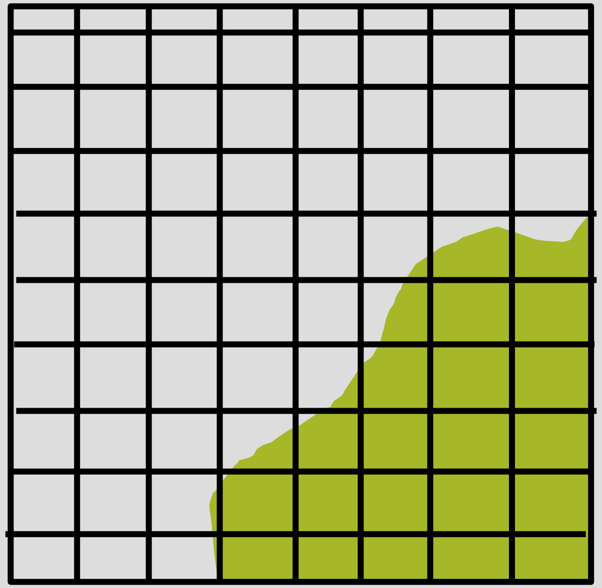
- Generate a cartesian grid without regard to the solid body
- Incorporate boundary conditions by modifying the equations near the boundary
- Allows discretization of complex domains without coordinate transformations or complicated discretization

Common Alternative



Grid generated to conform to solid body

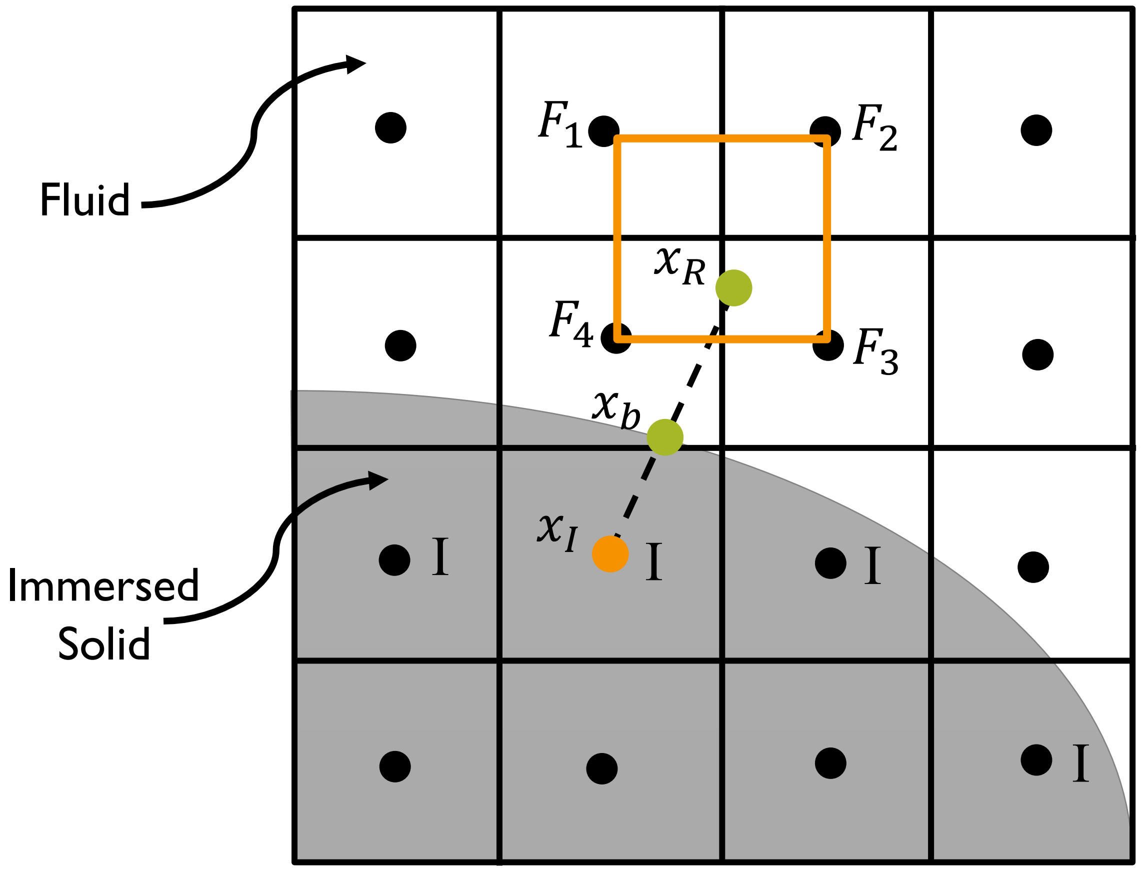
IBM



Grid generated regardless of immersed object

METHODOLOGY:

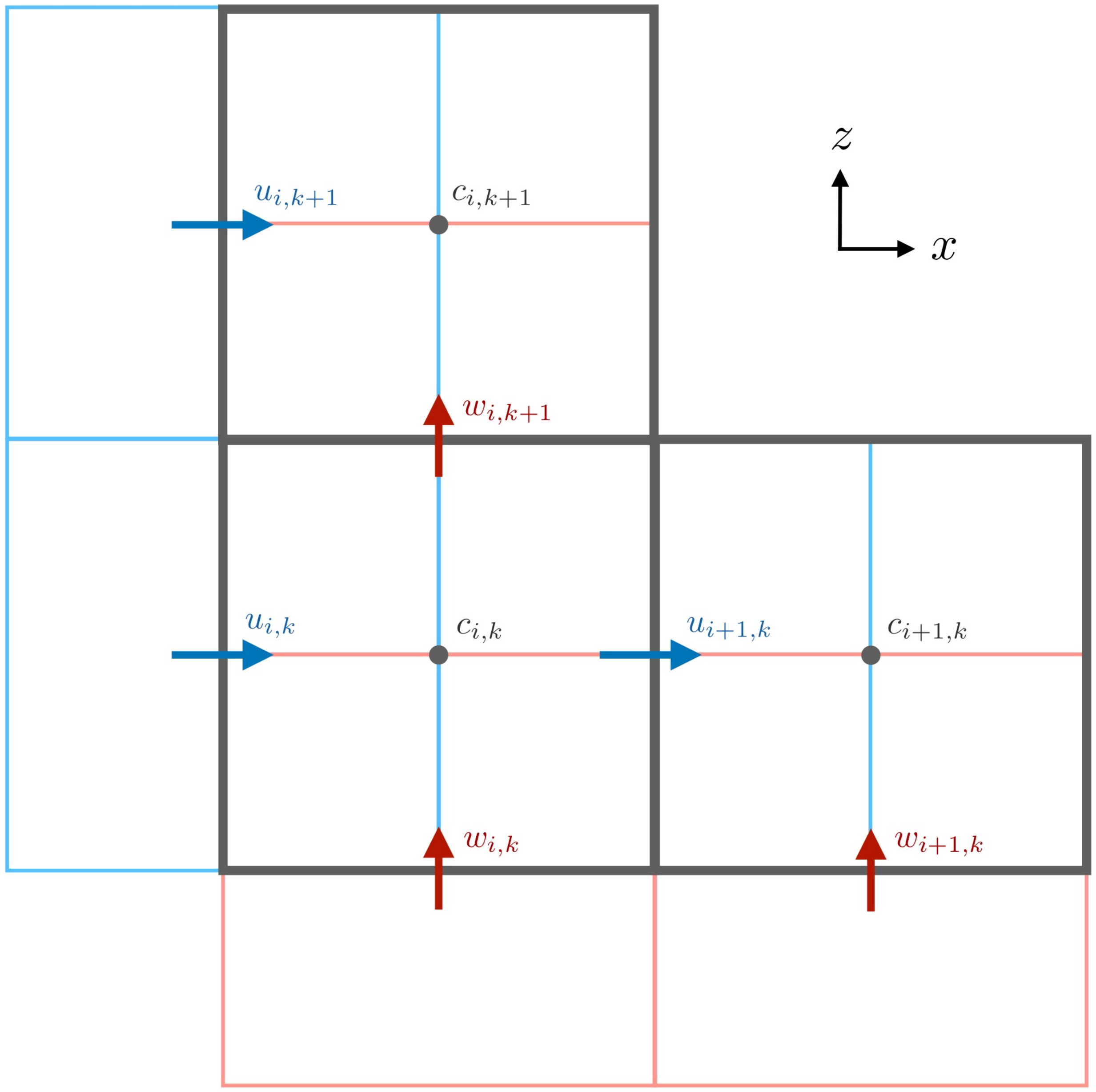
REFLECTION AND INTERPOLATION AT BOUNDARIES



Nasr-Azadani and Meiburg, (2011)

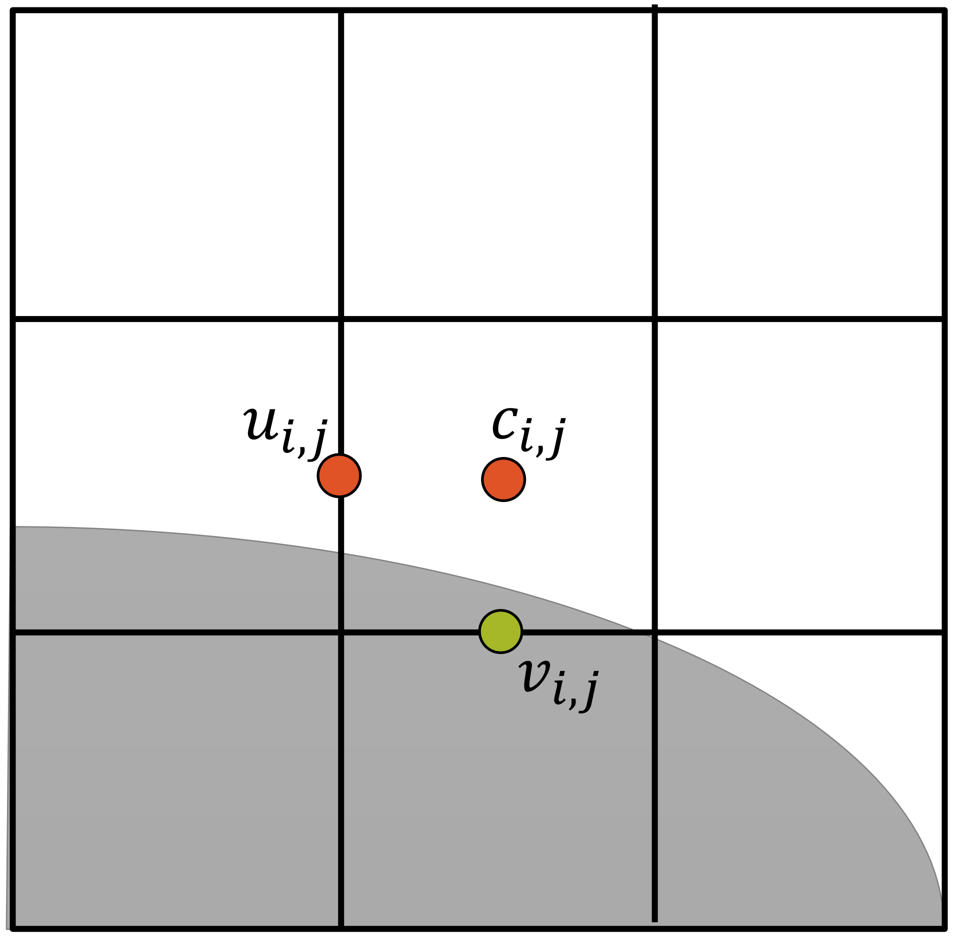
METHODOLOGY:

IBM WITH A STAGGERED GRID



METHODOLOGY:

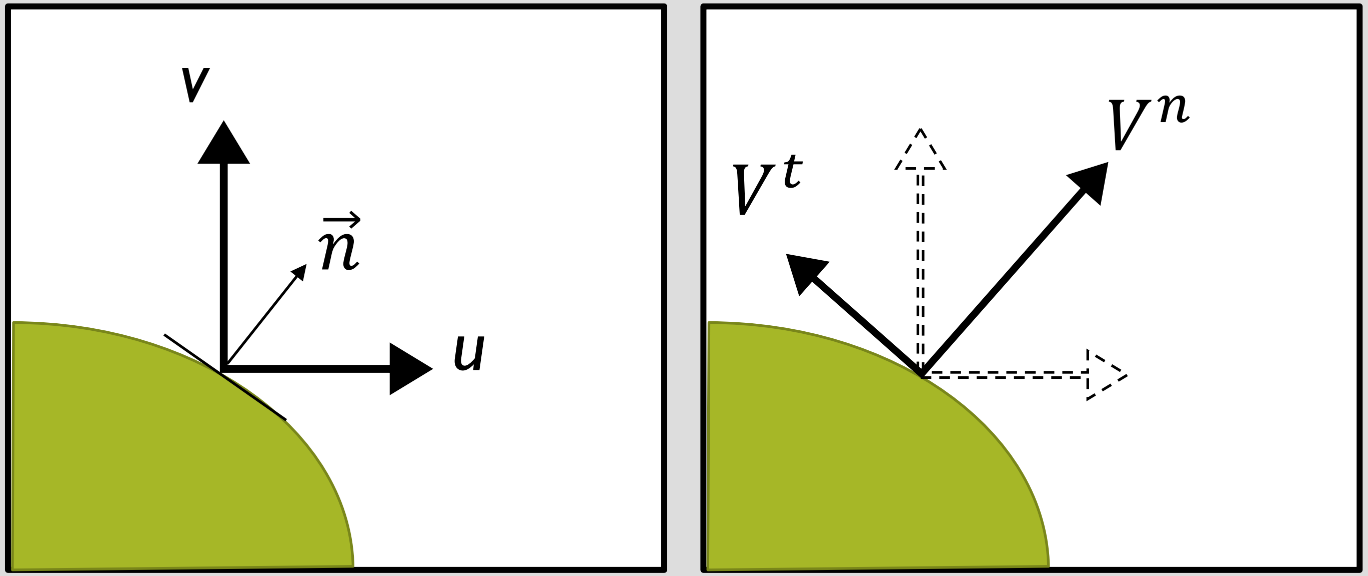
INDEPENDENTLY DONE FOR EACH VELOCITY COMPONENT



Nasr-Azadani and Meiburg, (2011)

METHODOLOGY: PROJECTING VELOCITIES INTO NORMAL AND TANGENTIAL COMPONENTS

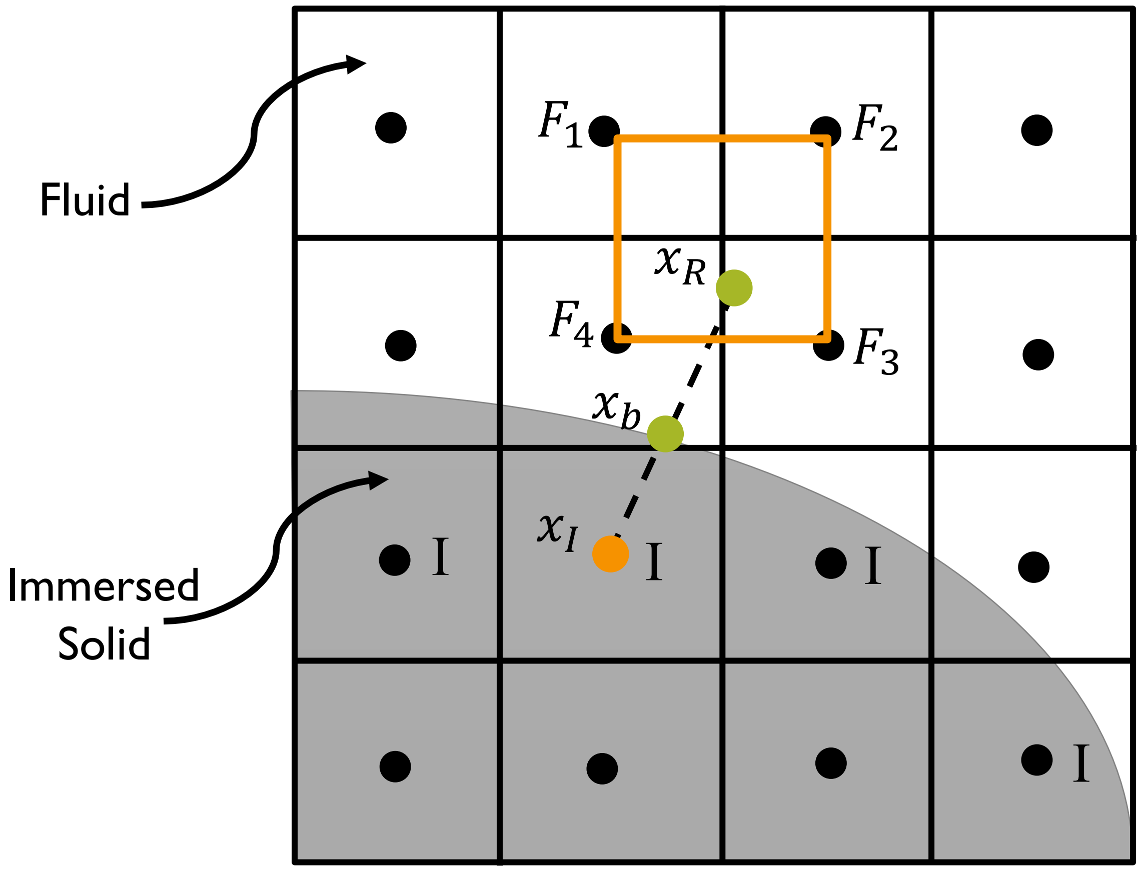
- Find tangential components from the surface normal, $\vec{n}$
- Interpolate for $< u, v, w >$
- Project velocity into tangential and normal components $< V^{t1}, V^{t2}, V^{n} >$



$\boldsymbol{U} = < u, v, w >$ $\boldsymbol{U} = < V^{t1}, V^{t2}, V^{n} >$

METHODOLOGY:

REFLECTION AND INTERPOLATION AT BOUNDARIES



Nasr-Azadani and Meiburg, (2011)

IMPLEMENTATION: OCEANANIGANS.JL WITHOUT IBM

- Time integral of momentum equation with pressure decomposition

$$\boldsymbol{U}^{n+1} - \boldsymbol{U}^{n} = \int_{t_n}^{t_{n+1}} \left[-\nabla\phi_{non} - \nabla\phi_{hyd} - (\boldsymbol{U}\cdot\nabla)\mathbf{U} - \mathbf{f}\times\boldsymbol{U} + \nabla\cdot\boldsymbol{\tau} + \boldsymbol{F_u} \right] dt$$

$$\boldsymbol{U}^{n+1} - \boldsymbol{U}^{n} \approx -\Delta t\, \nabla\phi_{non}^{n+1} + \int_{t_n}^{t_{n+1}} \boldsymbol{G_u}\, dt$$

- 3rd order Runge-Kutta with pressure correction, at kth stage:

$$\boldsymbol{U}^{k} = \boldsymbol{U}^{k} + \gamma^{k}\Delta t\boldsymbol{G}^{k} + \zeta^{k}\Delta t\boldsymbol{G}^{k-1}$$

$$\boldsymbol{U}^{k} = \boldsymbol{U}^{k} - \nabla\phi_{non}\Delta t(\gamma^{k} + \zeta^{k})$$

Souza, (2020)
Yinnian and Li, (2009)
Le and Moin, (1990)

IMPLEMENTATION: OCEANANIGANS.JL WITH IBM

- Time integral of momentum equation with pressure decomposition

$$\boldsymbol{U}^{n+1} - \boldsymbol{U}^{n} = \int_{t_n}^{t_{n+1}} \left[-\nabla\phi_{non} - \nabla\phi_{hyd} - (\boldsymbol{U}\cdot\nabla)\mathbf{U} - \mathbf{f}\times\boldsymbol{U} + \nabla\cdot\boldsymbol{\tau} + \boldsymbol{F}_{\boldsymbol{u}} \right] dt$$

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- 3rd order Runge-Kutta with pressure correction, at kth stage:

$$\boldsymbol{U}^{k} = \boldsymbol{U}^{k} + \gamma^{k}\Delta t\boldsymbol{G}^{k} + \zeta^{k}\Delta t\boldsymbol{G}^{k-1}$$

IBM Forcing

$$\boldsymbol{U}^{k} = \boldsymbol{U}^{k} - \nabla\phi_{non}\Delta t(\gamma^{k} + \zeta^{k})$$

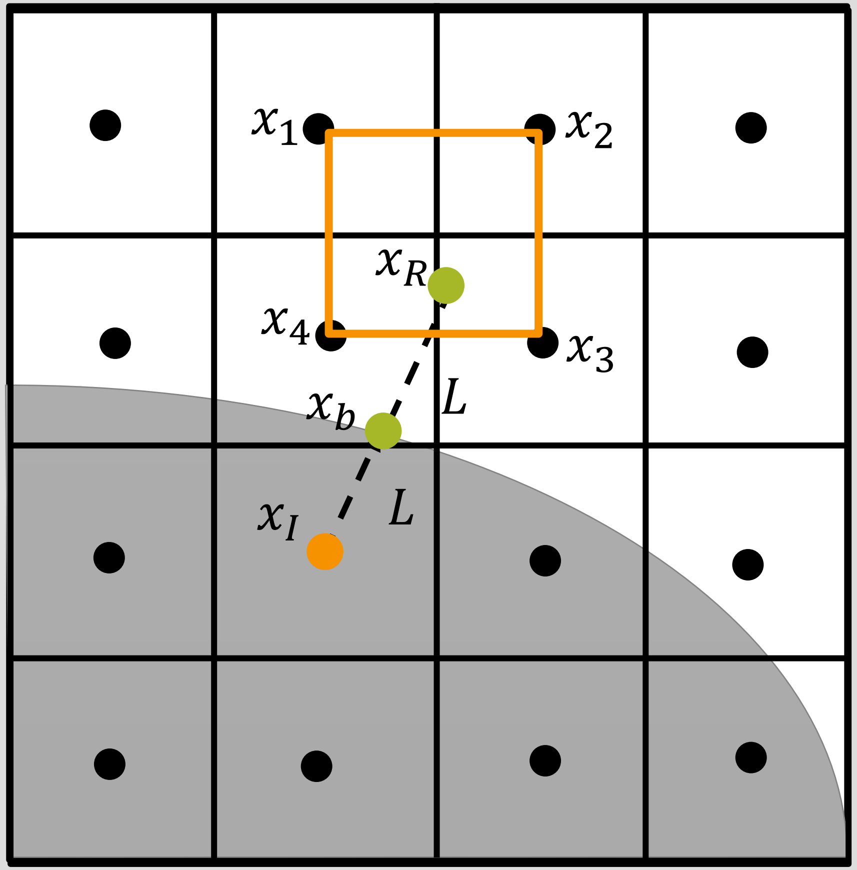
Souza, (2020)
Yinnian and Li, (2009)
Le and Moin, (1990)

IMPLEMENTATION:
IBM BOUNDARY INTERPOLATION

- Trilinear interpolation at reflected point x 3

$$q_R = \sum_{j=1}^{N} w_j q_j$$

- Projection: $\widehat{q_R}$
- Use $\widehat{q_R}$ and boundary condition to interpolate to node, I:
  - *DIRICHLET* $\quad \hat{q_I} = 2q_b - \widehat{q_R}$
  - *NEUMANN* $\quad \hat{q_I} = -2L \frac{\partial q}{dn}|_b + \widehat{q_R}$
- Project Back: q_I
- Correct velocity at x_I with component of q_I



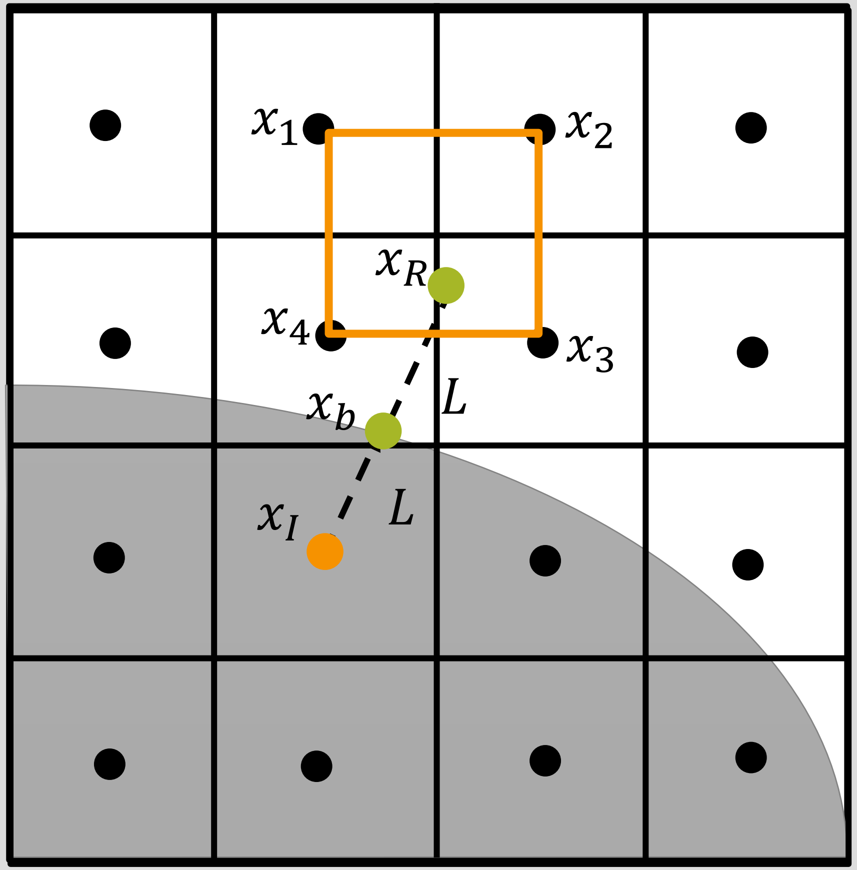
Nasr-Azadani and Meiburg, (2011)

IMPLEMENTATION:
IBM INTERPOLATION FOR TRACERS

- Tracers: temperature, salinity, dye, oxygen, buoyancy

$$\partial_t c = -u \cdot \nabla c \; - U \cdot \nabla c \; - u \cdot \nabla C - \nabla \cdot q_c + F_c$$

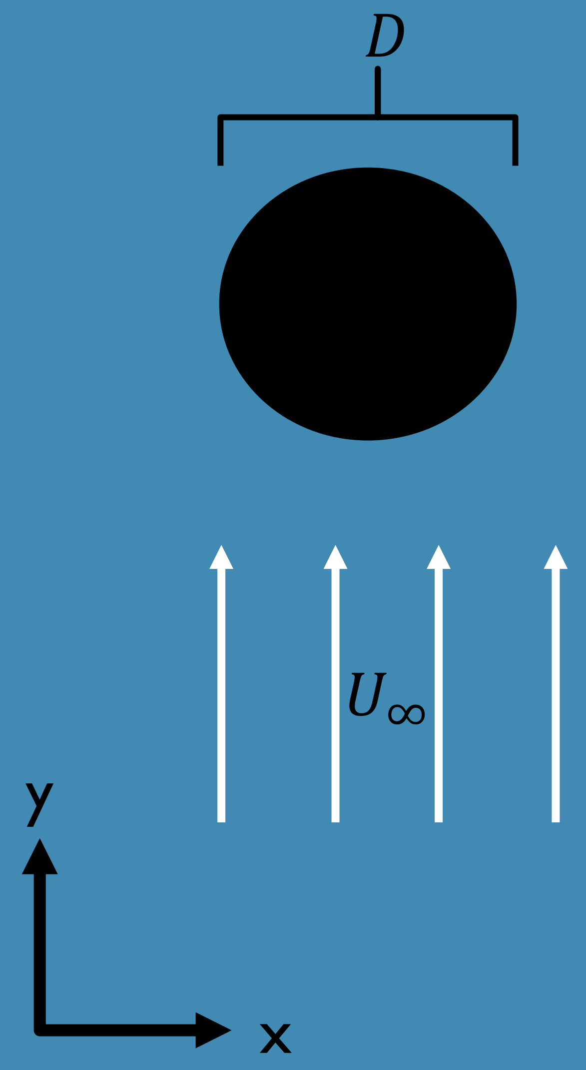
- Trilinear interpolation at reflected point
- Use c_R and boundary condition to interpolate to node, I
- Correct tracer at x_I with c_I



Nasr-Azadani and Meiburg, (2011)

VALIDATION: FLOW AROUND A CYLINDER

- Uniform flow in vertical, $U_\infty = 1$
- Domain $L_x \times L_y = 20 \times 30$
- Cylinder with diameter, $D = 2$, centered at $(x_c, y_c) = (30, 20\ )$
- Steady flow determined by Reynolds number, $Re_D = \frac{U_\infty D}{\nu} = 40$



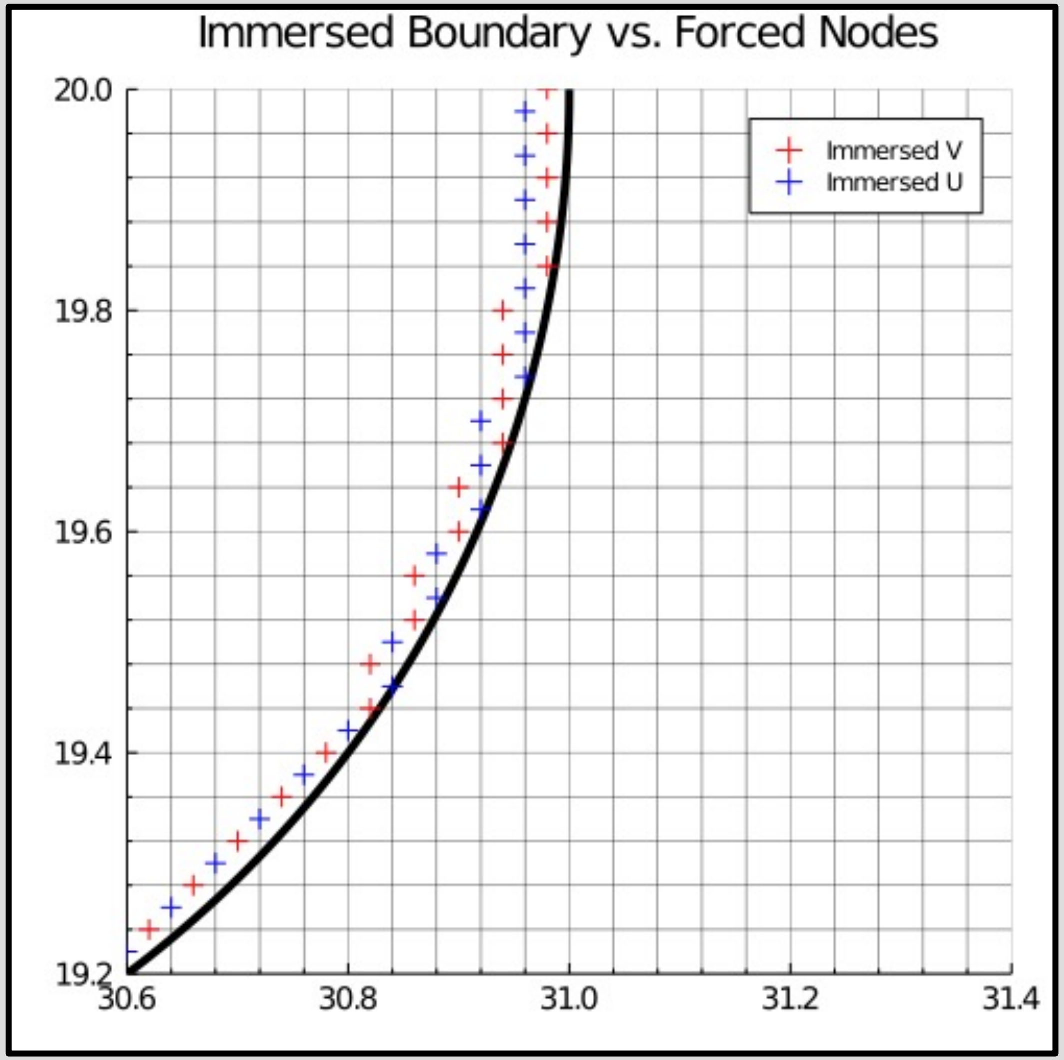
CASE STUDY:
UNIFORM FLOW AROUND CYLINDER

- Resolution:

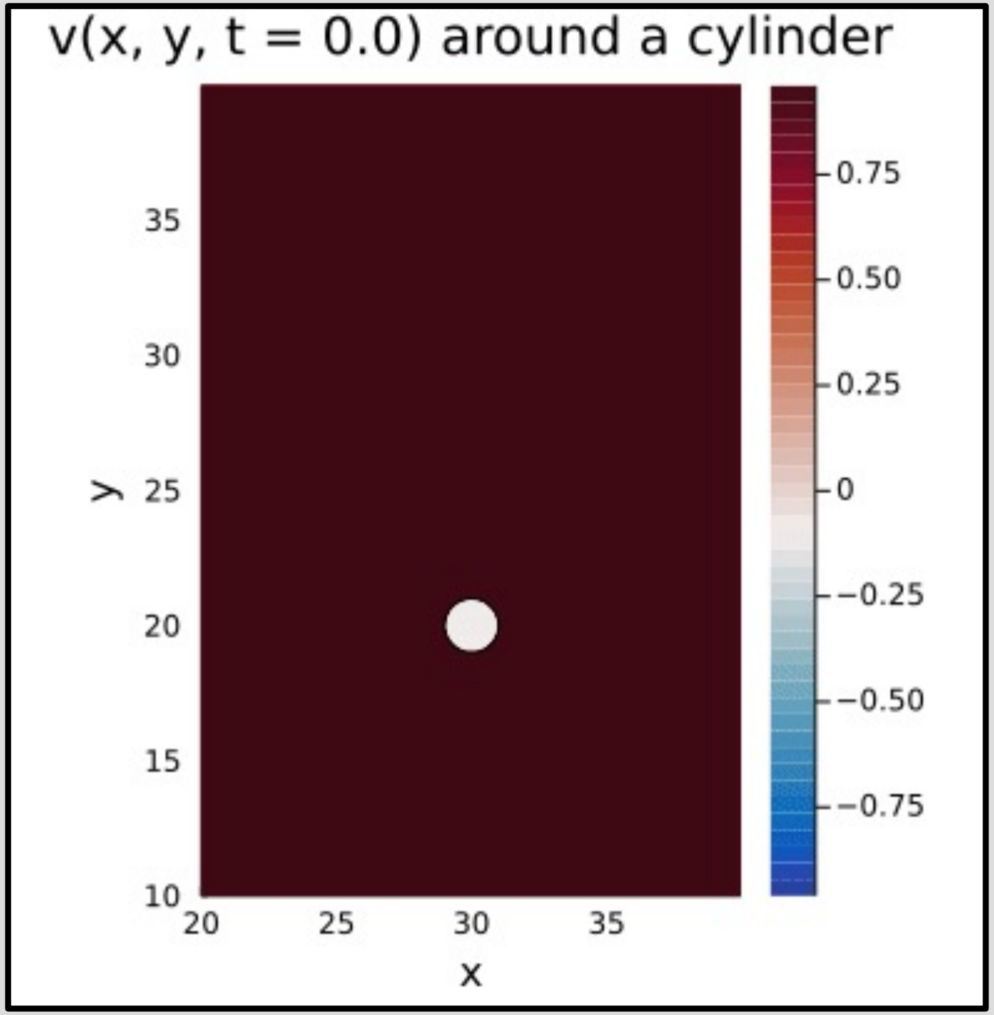
$\Delta x = \Delta y$	0.04	0.02
nx	500	1000
ny	750	1500
Δt	.0057	.0014

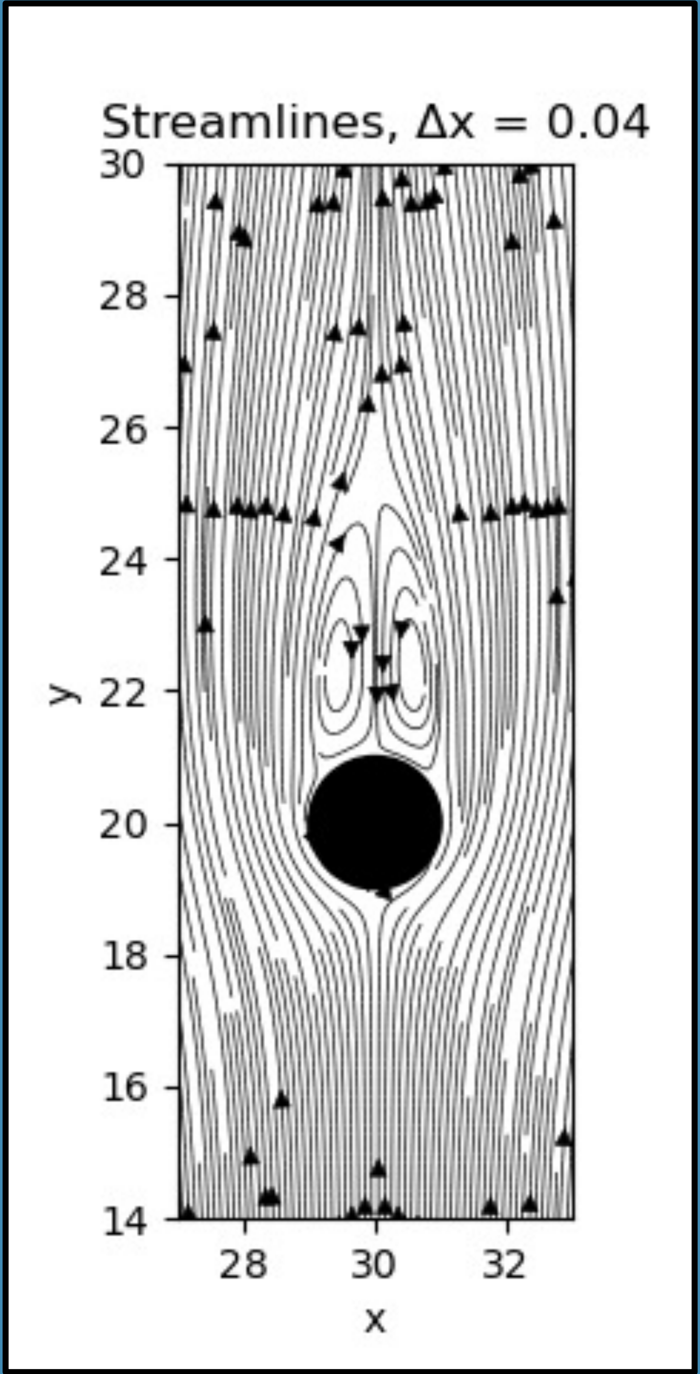
- CPU Time:

$\Delta x = \Delta y$	0.04	0.02
simulation s/min	1.5×10^{-2}	4.6×10^{-4}



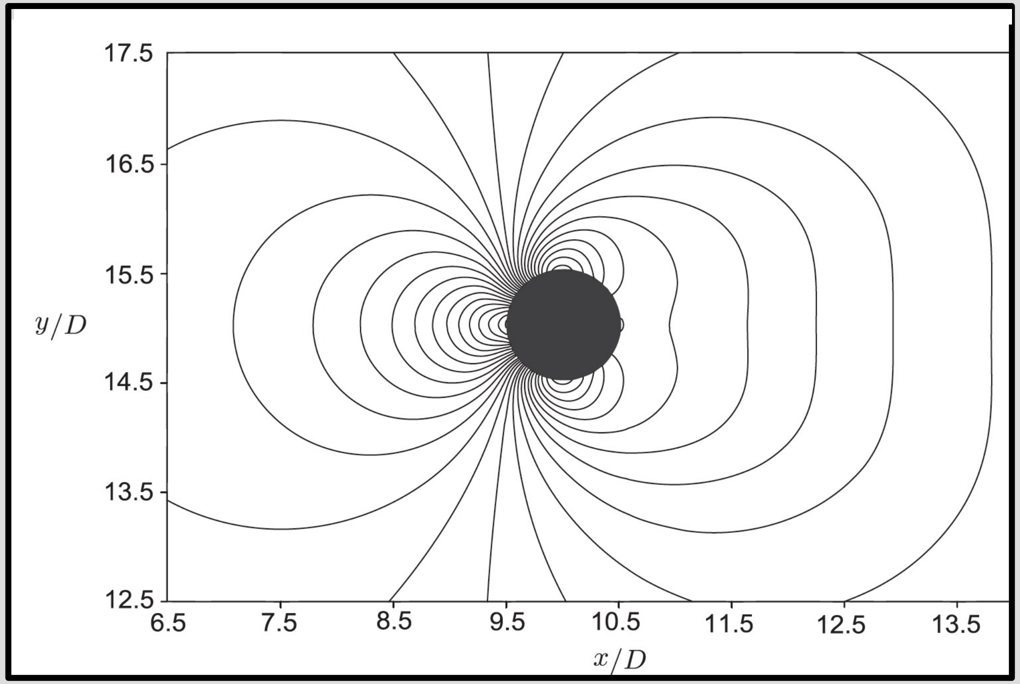
RESULTS: VELOCITY OF STEADY STATE FLOW





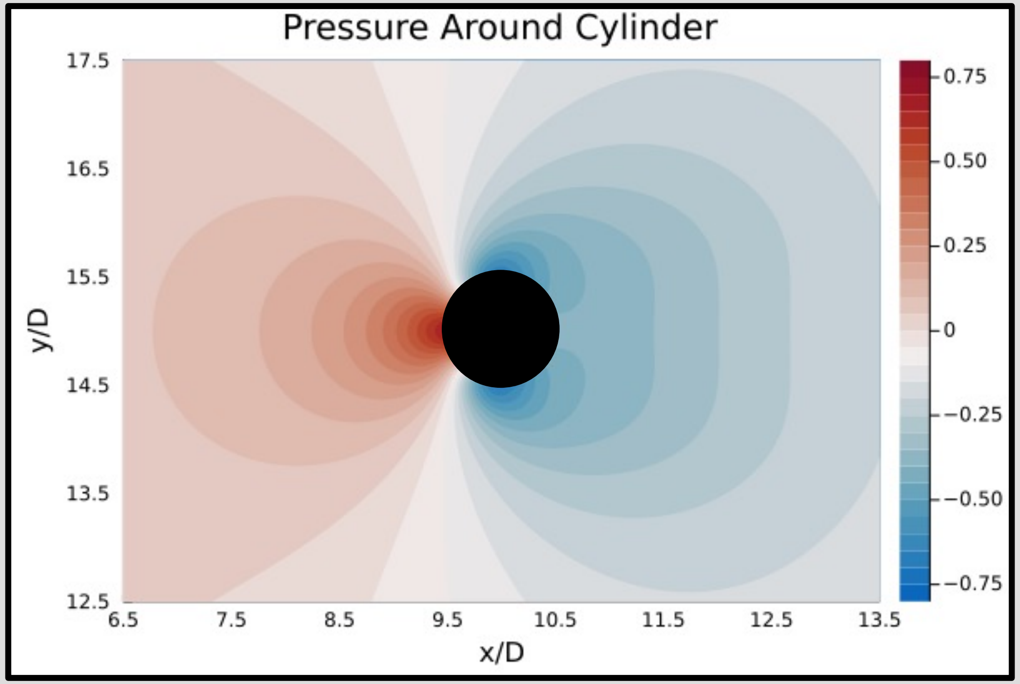
RESULTS:
STEADY-STATE PRESSURE CONTOURS

Nasr-Azadani and Meiburg Results



Average Front Stagnation Pressure: 0.571

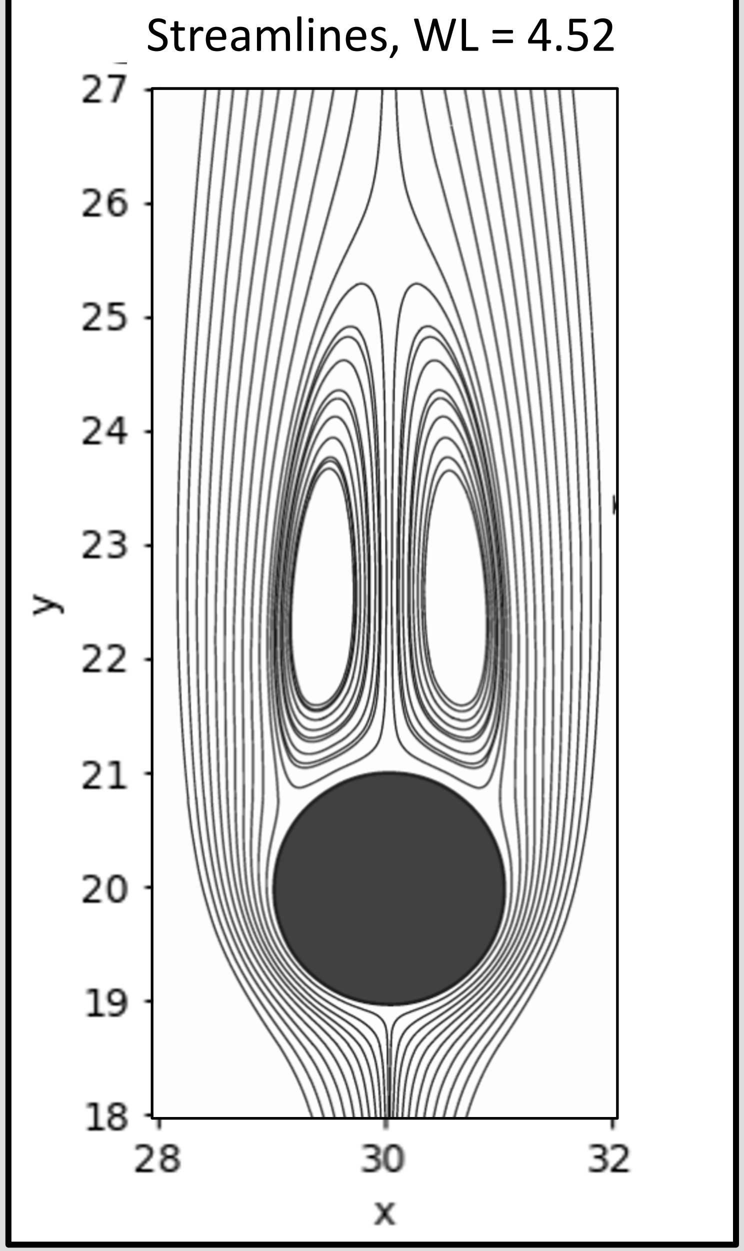
Current Study Results



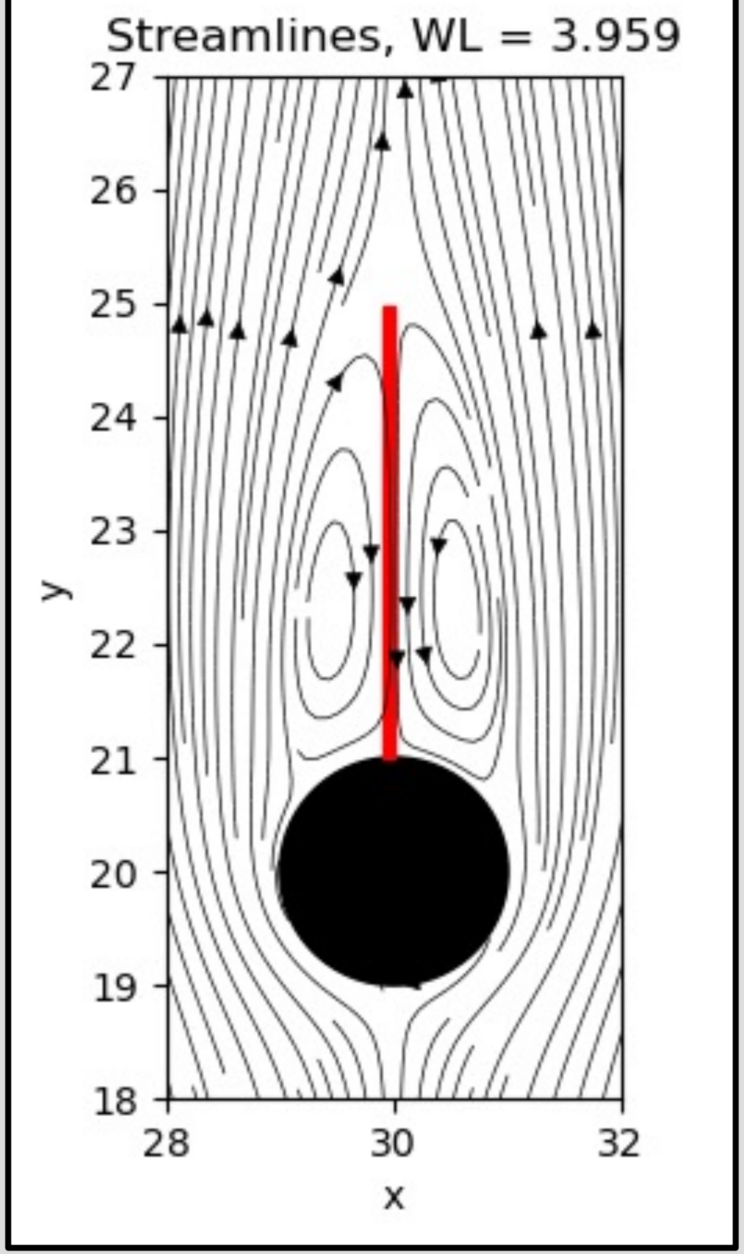
Front Stagnation Pressure: 0.585

Nasr-Azadani and Meiburg, (2010)

RESULTS: WAKE LENGTH



Nasr-Azadani and Meiburg, (2010)



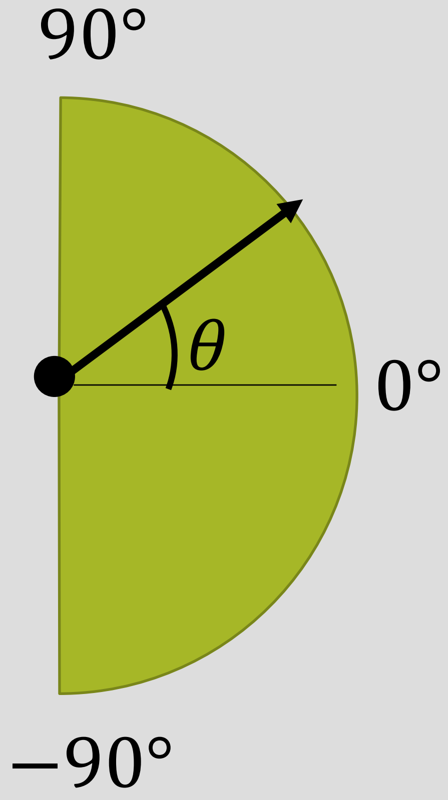
Current Study

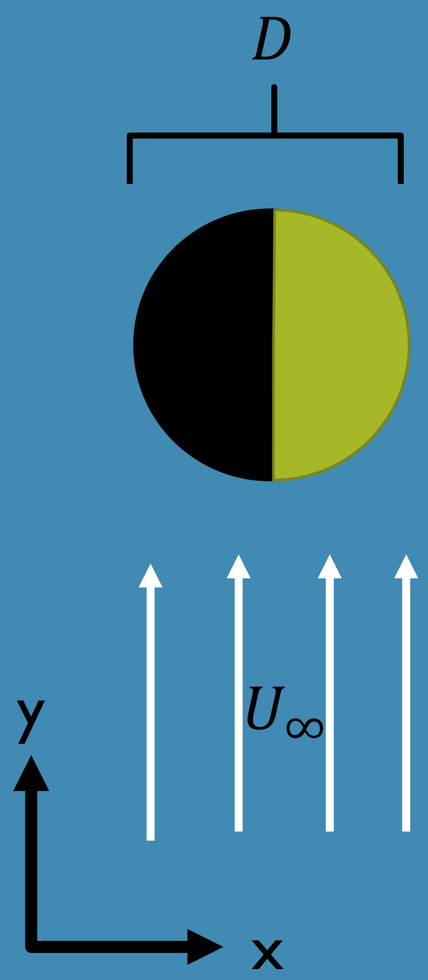
Study	$\frac{WL}{D}$
Current, $\Delta x = 0.04$	1.98
Fornberg	2.24
Nasr-Azadani et al.	2.26
Takami and Keller	2.33
Dennis and Chang	2.35

RESULTS:
ON THE SURFACE OF A CYLINDER

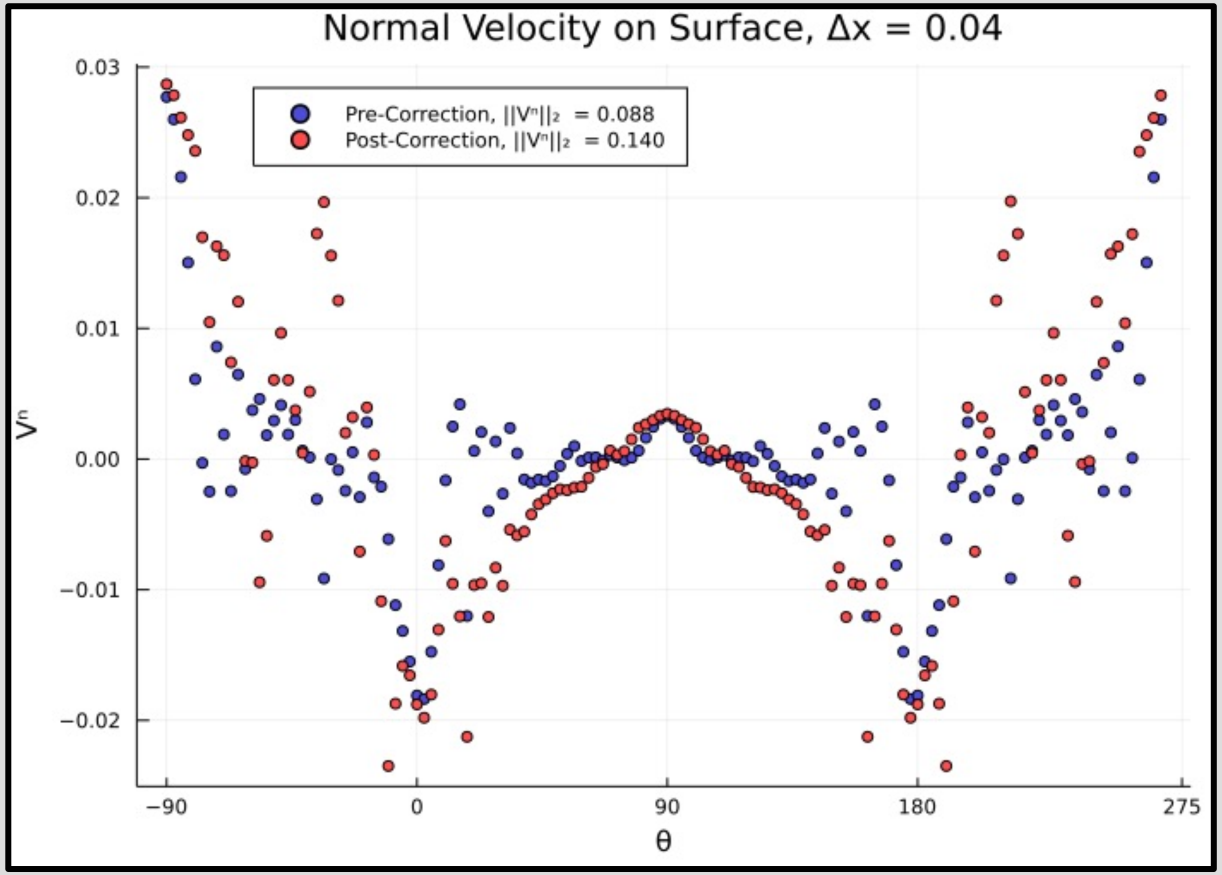
- Uniform flow is symmetrical about the freestream direction

- We can look at results on this half-surface at positions measured by the angle, θ





RESULTS:
NORMAL VELOCITY ON SURFACE OF THE CYLINDER



Study	$\| V_n - 0 \|_2$
Before Pressure Correction	0.088
After Pressure Correction	0.140

RESULTS:
PRESSURE COEFFICIENT ON SURFACE OF THE CYLINDER

- Pressure Coefficient

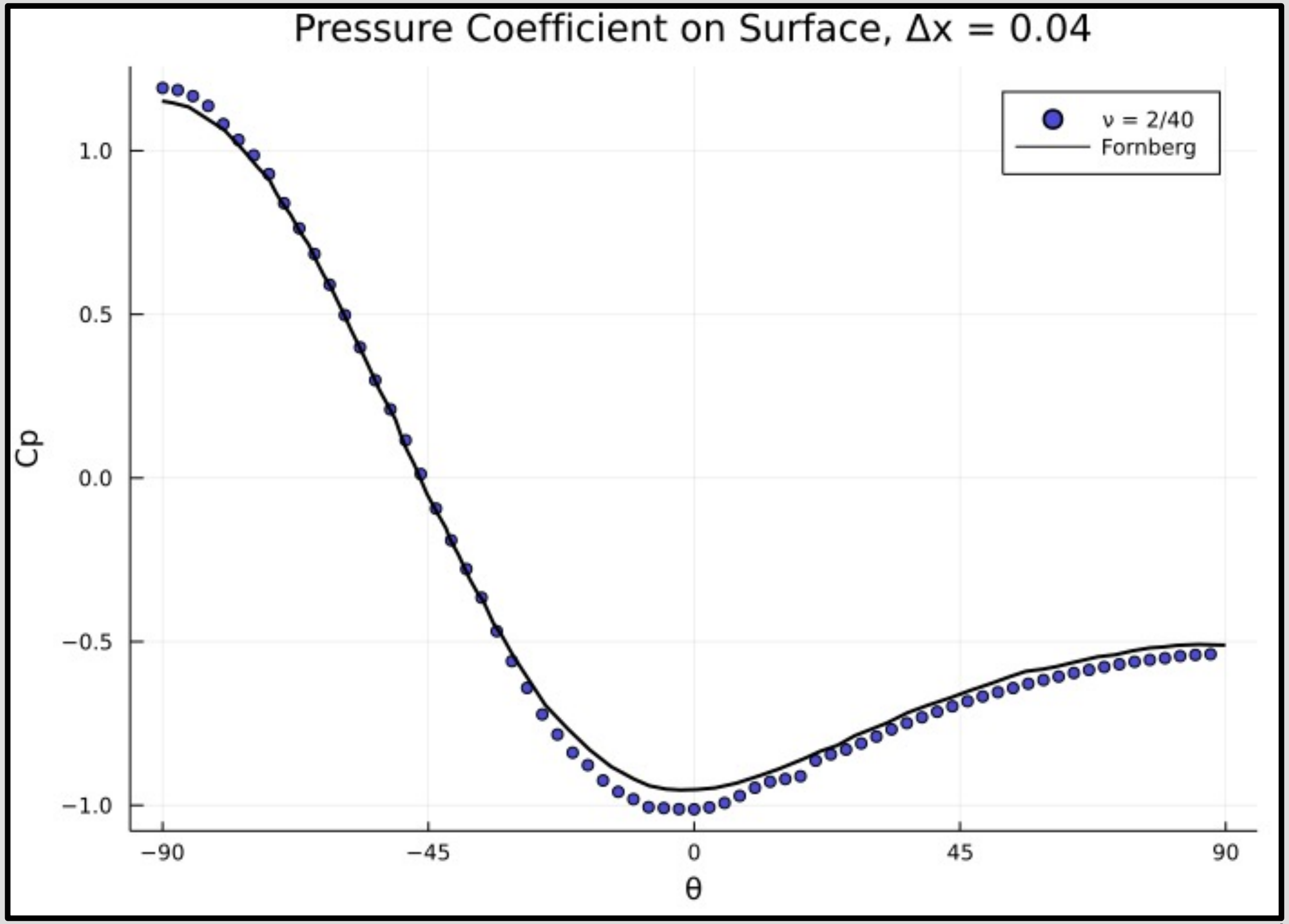
$$C_p = \frac{p - p_\infty}{0.5\rho U_\infty^2}$$

– p_∞, freestream pressure

– ρ, fluid density

– U_∞, freestream velocity

– p, pressure on cylinder



Fornberg, (1980)

RESULTS:
FRICTION COEFFICIENT ON SURFACE OF THE CYLINDER ARBITRARY BOUNDARIES

- Friction Coefficient

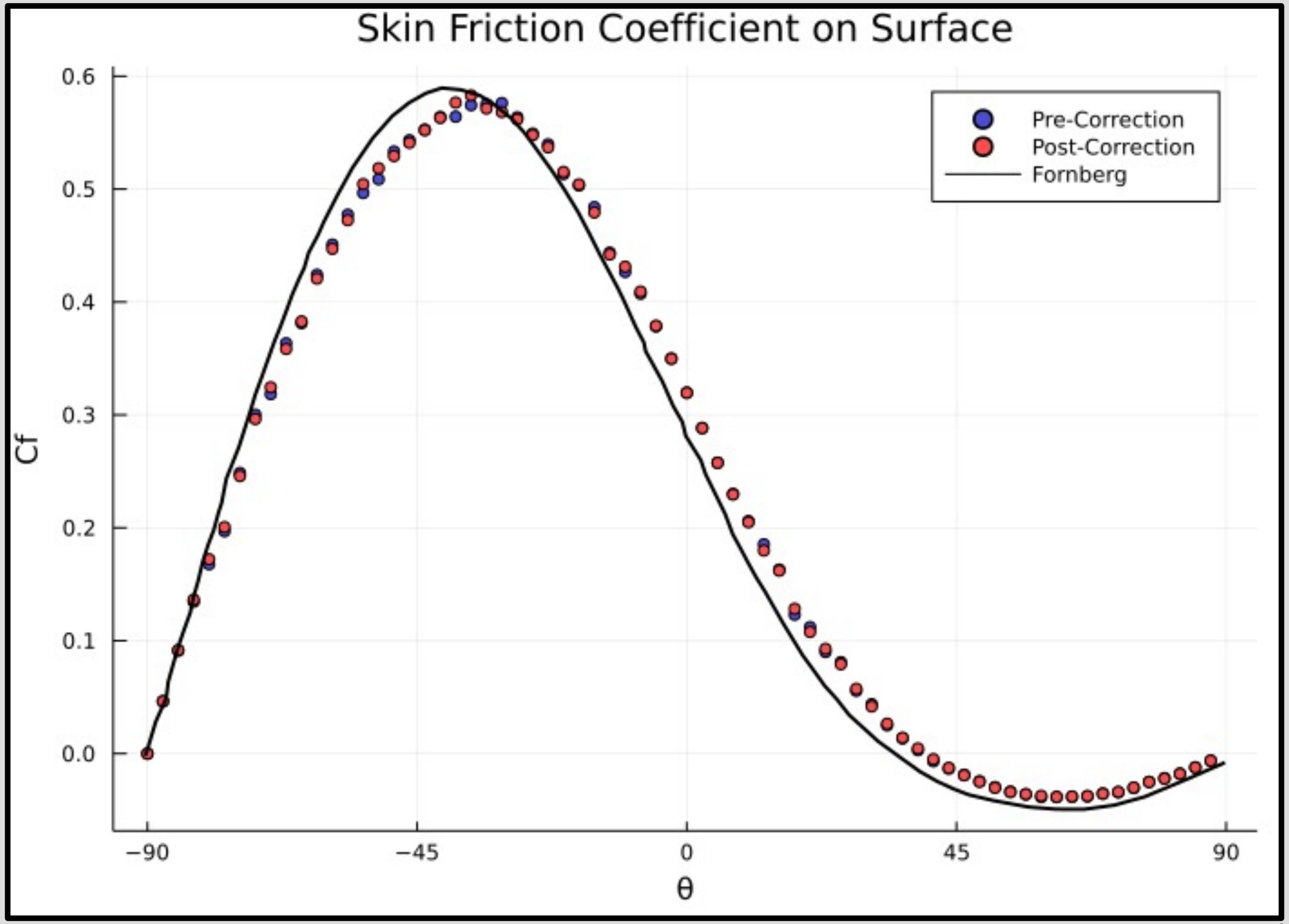
$$C_f = \frac{\tau_w}{0.5\rho U_\infty^2} = 2\nu \frac{\partial V_t}{\partial n}$$

– τ_w, stress at the "wall"

– ρ, fluid density

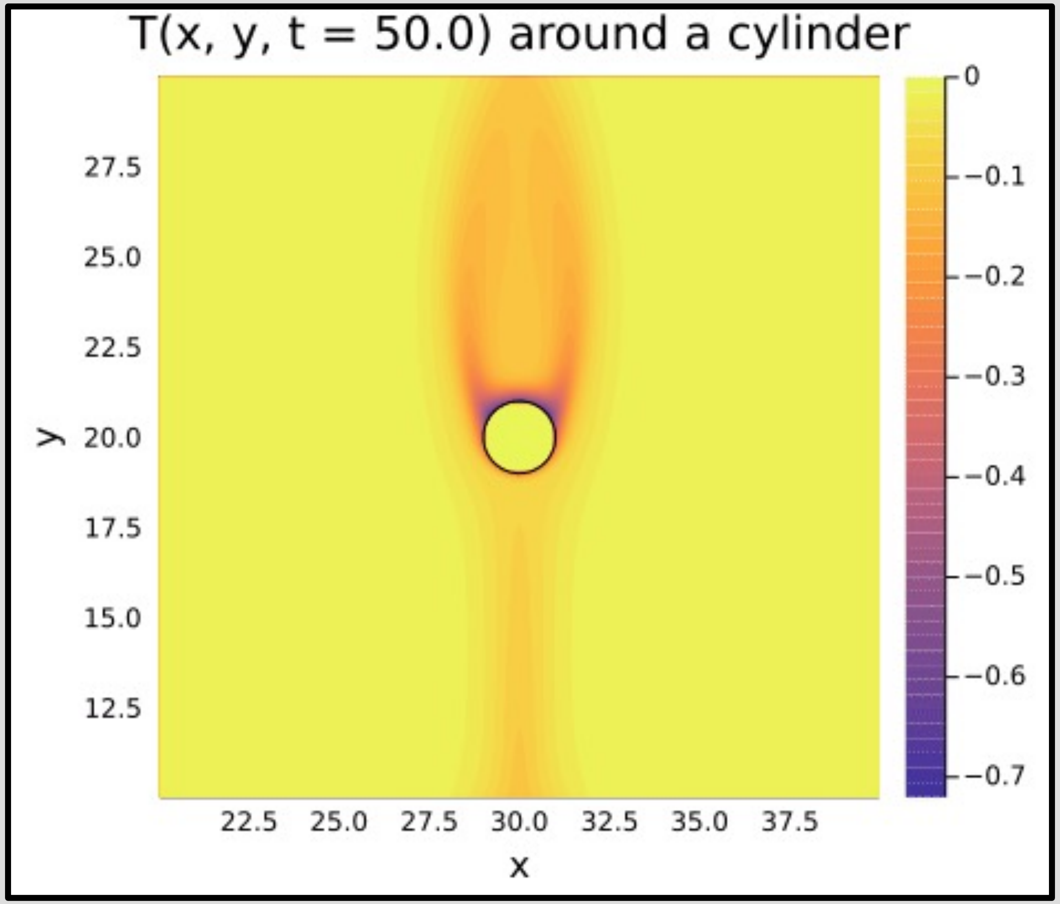
– U_∞, freestream velocity

– ν, kinematic viscosity



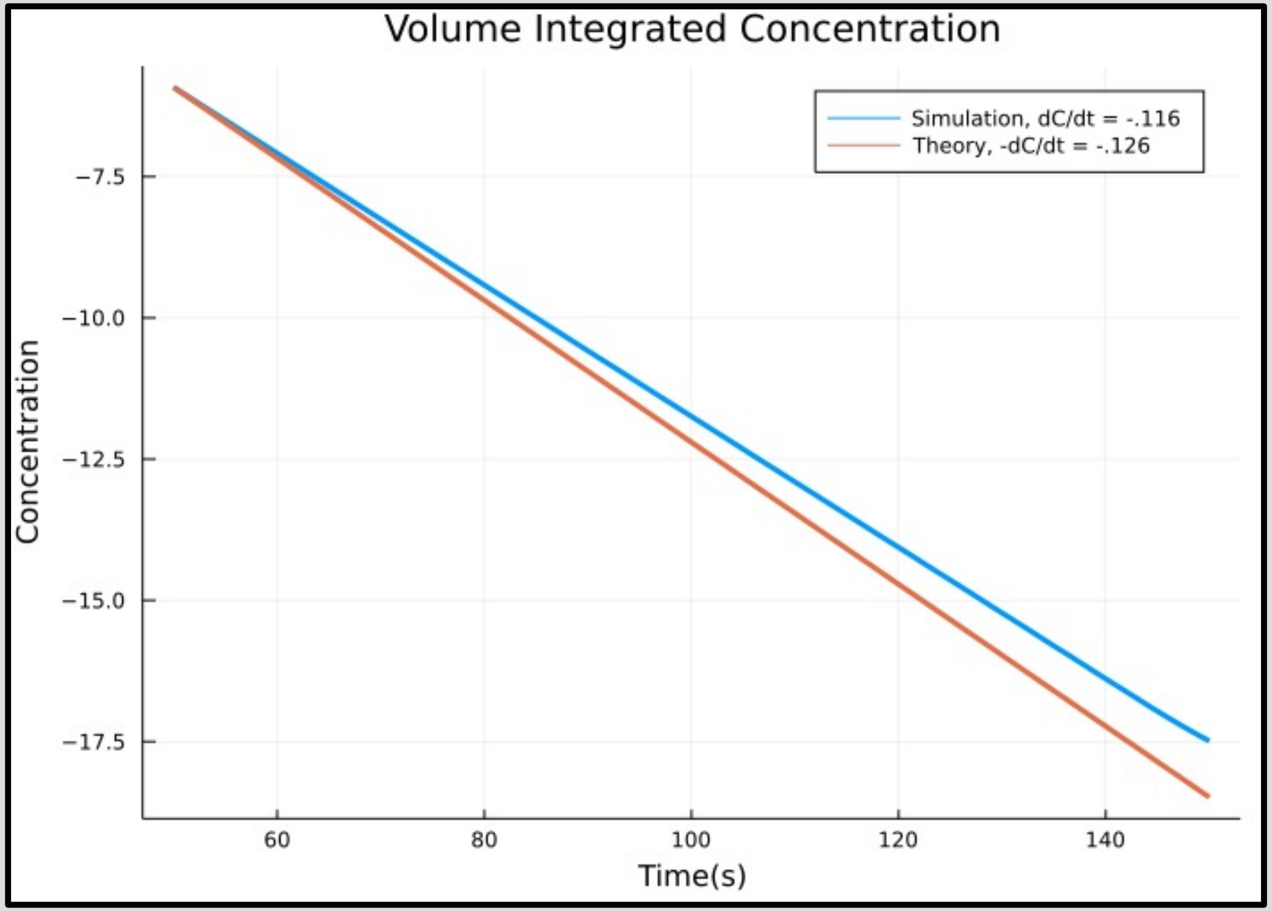
Fornberg, (1980)

CASE STUDY: TRACER WITH NEUMANN BOUNDARY CONDITIONS AROUND A CYLINDER



- $\mathrm{Re} = 100$
- $350\ x\ 350$ grid points
- $\frac{\partial c}{\partial n} = 1$ on cylinder boundary
- Set fluid concentration to $C = 0$

RESULTS: CONCENTRATION RATE IN FLUID



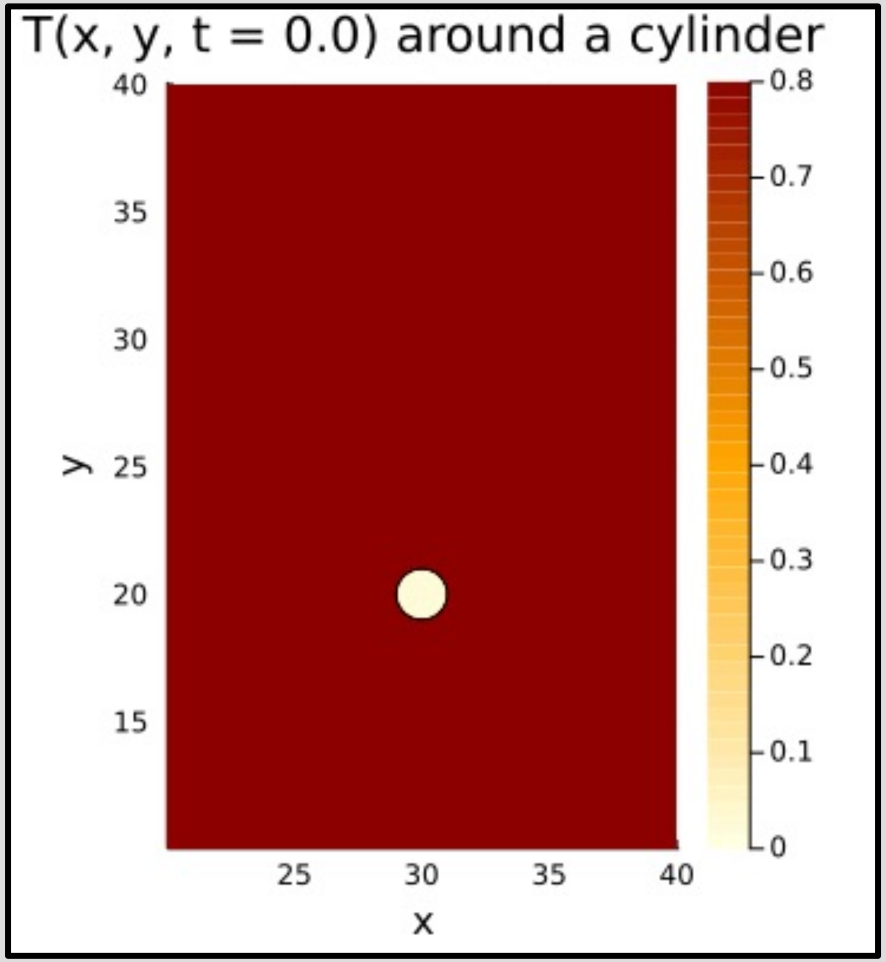
- Cylinder should leak concentration into the fluid at a rate of

$$\frac{dC}{dt} = \kappa(\pi D)\frac{\partial C}{\partial n} \approx 0.126$$

- For this simulation

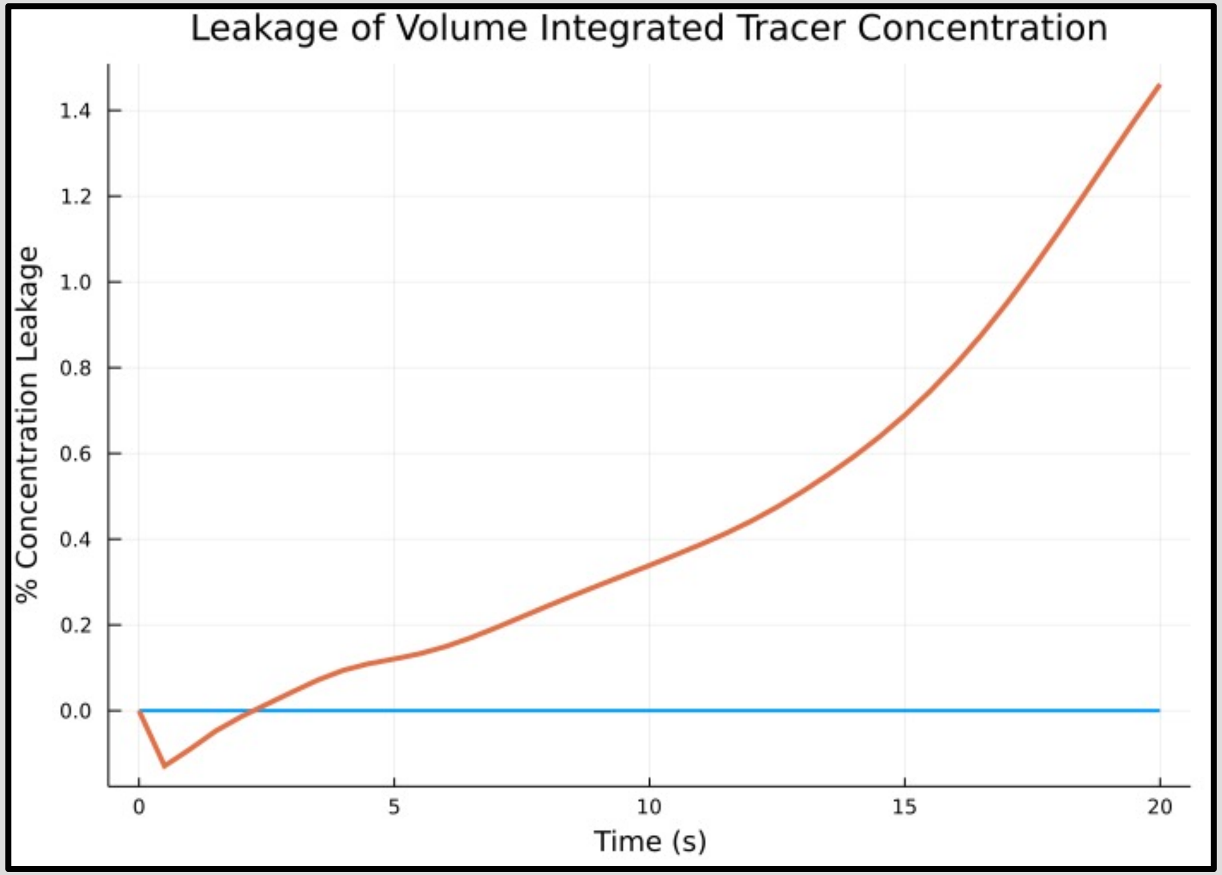
$$\frac{dC}{dt} = -0.116$$

CASE STUDY: TRACER WITH NEUMANN BOUNDARY CONDITIONS AROUND A CYLINDER



- $\mathrm{Re} = 40$
- $500\ x\ 750$ grid points
- $\frac{\partial c}{\partial n} = 0$ on cylinder boundary
- Set fluid concentration to $C = 1$

RESULTS:
CONCENTRATION LEAKAGE IN FLUID



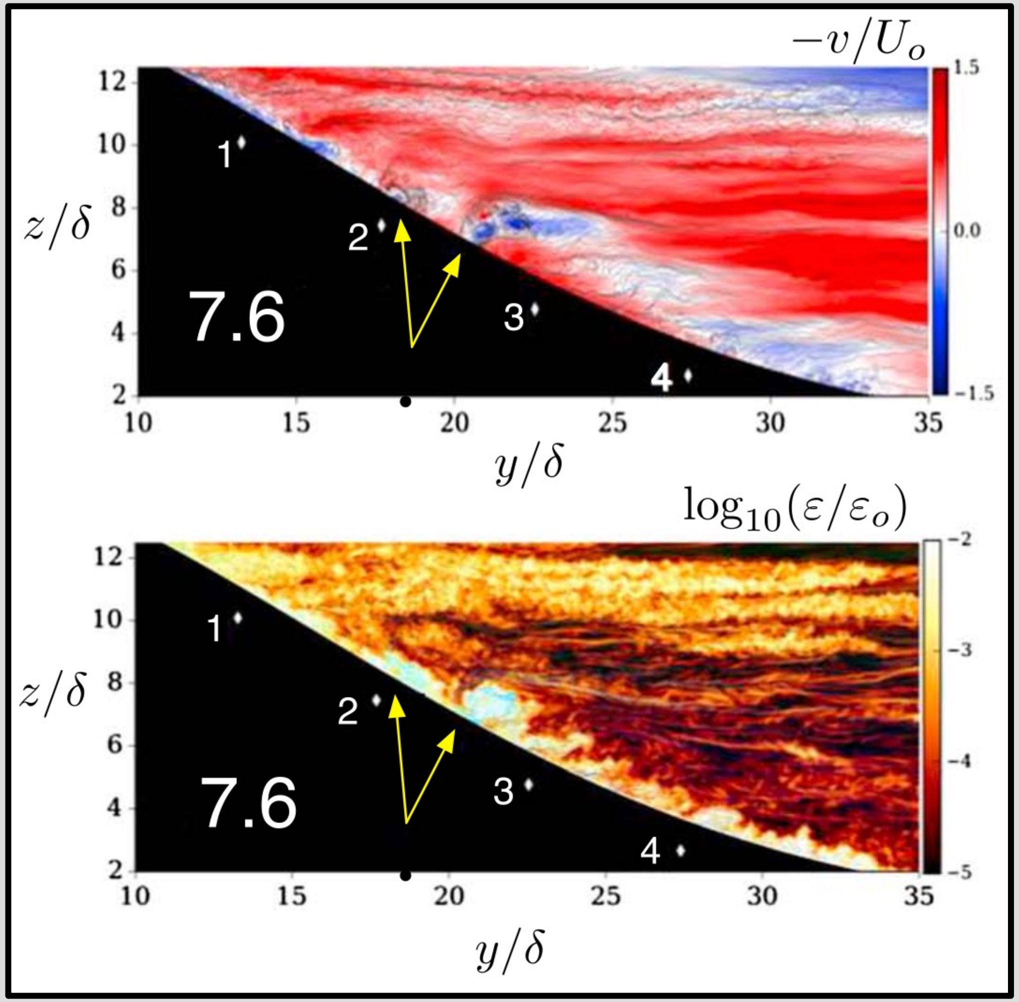
- Cylinder concentration should remain steady in time, with no leakage
- After 20s there is a 1.4% increase in the concentration of tracer in the fluid
- Leakage of tracer much less than the velocity is leaking

IMMERSED BOUNDARY METHOD FEATURES

- Able to work with arbitrary boundaries
- Handles Dirichlet and Neumann boundary conditions
- Works with both momentum and tracer equations.

APPLYING IBM ON OCEAN PROBLEMS

- Breaking of internal waves approaching a shoaling coast
- No-slip immersed bottom boundary
- Causes near-boundary fluid to move into interior



Winters, (2015)

FUTURE WORK

- Speed things up and make more “user friendly”
- Look into the impact of projection method
- Test on geophysical applications
- Add final IBM implementation to Oceananigans.jl release

SOURCES

- Arthur, Robert S., et al. "Evaluating Implementations of the Immersed Boundary Method in the Weather Research and Forecasting Model." *Monthly Weather Review*, vol. 148, no. 5, 2020, pp. 2087–2109., doi:10.1175/mwr-d-19-0219.1.
- Balaras, Elias. "Modeling Complex Boundaries Using an External Force Field on Fixed Cartesian Grids in Large-Eddy Simulations." *Computers & Fluids*, vol. 33, 2004, pp. 375–404.
- E. Griffith, Boyce, and Xiaoyu Luo. "Hybrid Finite Difference/Finite Element Immersed Boundary Method." *International Journal for Numerical Methods in Biomedical Engineering*, vol. 33, no. 12, 2017, doi:10.1002/cnm.2888.
- Eymard, Robert, and Thierry Gallouã. "Finite Volume Method." *Scholarpedia*, vol. 5, no. 6, 2010, p. 9835., doi:10.4249/scholarpedia.9835.
- Fadlun, E.A., et al. "Combined Immersed-Boundary Finite-Difference Methods for Three-Dimensional Complex Flow Simulations." *Journal of Computational Physics*, vol. 161, no. 1, 2000, pp. 35–60., doi:10.1006/jcph.2000.6484.
- Guermond, J.l., et al. "An Overview of Projection Methods for Incompressible Flows." *Computer Methods in Applied Mechanics and Engineering*, vol. 195, no. 44-47, 2006, pp. 6011–6045., doi:10.1016/j.cma.2005.10.010.
- Gula, Jonathan, et al. "Topographic Generation of Submesoscale Centrifugal Instability and Energy Dissipation." *Nature Communications*, vol. 7, no. 1, 2016, doi:10.1038/ncomms12811.
- He, Yinnian, and Jian Li. "Numerical Implementation of the Crank-Nicolson/Adams-Bashforth Scheme for the Time-Dependent Navier-Stokes Equations." *International Journal for Numerical Methods in Fluids*, 2009, doi:10.1002/fld.2035.
- Iaccarino, Gianluca, and Roberto Verzicco. "Immersed Boundary Technique for Turbulent Flow Simulations." *Applied Mechanics Reviews*, vol. 56, no. 3, 2003, pp. 331–347., doi:10.1115/1.1563627.

SOURCES

- Kallemov, Bakytzhan, et al. "An Immersed Boundary Method for Rigid Bodies." *Communications in Applied Mathematics and Computational Science*, vol. 11, no. 1, 2016, pp. 79–141., doi:10.2140/camcos.2016.11.79.
- Le, Hung, and Parviz Moin. "An Improvement of Fractional Step Methods for the Incompressible Navier-Stokes Equations." *Journal of Computational Physics*, vol. 92, 1991, pp. 369–379., doi:0021-9991/91.
- Marshall, John, et al. "A Finite-Volume, Incompressible Navier Stokes Model for Studies of the Ocean on Parallel Computers." *Journal of Geophysical Research: Oceans*, vol.102, no. C3, 1997, pp. 5753–5766., doi:10.1029/96jc02775.
- Mcwilliams, James C. "Submesoscale Currents in the Ocean." *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 472, no. 2189, 2016, p. 20160117., doi:10.1098/rspa.2016.0117.
- M.H. Chung. Cartesian cut cell approach for simulating incompressible flows with rigid bodies of arbitrary shape. Computers & Fluids, 35(6):607{623, 2006. doi:10.1016/j.compfluid.2005.04.005.
- Mittal, Rajat, and Gianluca Iaccarino. "Immersed Boundary Methods." *Annual Review of Fluid Mechanics*, vol. 37, no. 1, 2005, pp. 239–261., doi:10.1146/annurev.fluid.37.061903.175743.
- Nasr-Azadani, M.M., and E. Meiburg. "TURBINS: An Immersed Boundary, Navier-Stokes Code for the Simulation of Gravity and Turbidity Currents Interacting with Complex Topographies." *Computers & Fluids*, vol. 45, 2011, pp. 14–28., doi:10.1016/j.compfluid.2010.11.023.
- Ramadhan, Ali, et al. "Oceananigans.jl." *CliMA/Oceananigans.jl*, GitHub, 2020, github.com/CliMA/Oceananigans.jl.
- Schumann, Ulrich, and Roland A Sweet. "Fast Fourier Transforms for Direct Solution of Poisson's Equation with Staggered Boundary Conditions." *Journal of Computational Physics*, vol. 75, no. 1, 1988, pp. 123–137., doi:10.1016/0021-9991(88)90102-7.

SOURCES

- Souza, Andre Nogueira, et al. "Uncertainty Quantification of Ocean Parameterizations: Application to the K-Profile-Parameterization for Penetrative Convection." *Journal of Advances in Modeling Earth Systems*, 2020, doi:10.1002/essoar.10502546.1.
- Verzicco, Roberto, et al. "Large Eddy Simulation in Complex Geometric Configurations Using Boundary Body Forces." *AIAA Journal*, vol. 38, 2000, pp. 427–433., doi:10.2514/3.14430.
- Wagner, Gregory, et al. "Near-Inertial Waves and Turbulence Driven by the Growth of Swell." *Journal of Physical Oceanography*, 30 July 2020, doi:10.1002/essoar.10503838.1.
- Wenegrat, Jacob O., and Leif N. Thomas. "Centrifugal and Symmetric Instability during Ekman Adjustment of the Bottom Boundary Layer." *Journal of Physical Oceanography*, vol. 50, no. 6, 2020, pp. 1793–1812., doi:10.1175/jpo-d-20-0027.1.
- Wenegrat, Jacob O., et al. "Submesoscale Baroclinic Instability in the Bottom Boundary Layer." *Journal of Physical Oceanography*, vol. 48, no. 11, 2018, pp. 2571–2592., doi:10.1175/jpo-d-17-0264.1.
- Winters, K. B. "Tidally driven mixing and dissipation in the stratified boundary layer above steep submarine topography." *Geophysical Research Letters*. AGUPublications, 2015.
- T. Ye, R. Mittal, H.S. Udaykumar, and W. Shyy. An Accurate Cartesian Grid Method for Viscous Incompressible Flows with Complex Immersed Boundaries. Journal of Computational Physics, 156(2):209{240, 1999. doi:10.1006/jcph.1999.6356.