Quiz 11, MATH 246, Professor Radu Balan, Tuesday, 1 December 2009

Your Name: ____

UMD SID:____

Discussion Instructor (circle one): Jeremy Schwartz Geetanjali Kachari

Discussion Time (circle one): 8:00 9:00 10:00 11:00

[2x5pts] For each of the following systems of 1st order differential equations:

a. [1pt] Find the eigenvalues and eigenvectors;
b. [2pts] Classify the critical point (0,0) as to type and determine whether it is stable/asymptotically stable or unstable;
c. [2pts] Sketch several trajectories in the phase plane.

1. $\frac{dx}{dt} = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} x$

<u>Solution</u>

a. Characteristic polynomial:

$$p_A(s) = \det(A - sI) = \begin{vmatrix} 3-s & -2 \\ 4 & -1-s \end{vmatrix} = s^2 - 2s + 5$$

Hence the eigenvalues are: $\lambda_{1,2} = 1 \pm 2i$

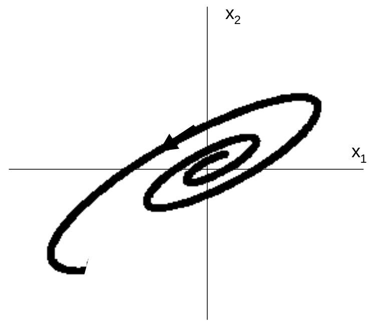
Eigenvectors:

$$\left(A - (1+2i)I\right)v^1 = 0 \Rightarrow \begin{pmatrix} 2-2i & -2 \\ 4 & -2-2i \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow v^1 = \begin{pmatrix} 1 \\ 1-i \end{pmatrix} a$$

$$\left(A - (1-2i)I\right)v^2 = 0 \Rightarrow \begin{pmatrix} 2+2i & -2 \\ 4 & -2+2i \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow v^2 = \begin{pmatrix} 1 \\ 1+i \end{pmatrix} a$$

b. The origin is an unstable spiral (or, spiral source).

c. Phase portrait: At the point (0,1) the vector is $A\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$



2. $\frac{dx}{dt} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} x$

<u>Solution</u>

a) Eigenvalues and eigenvectors:

Since A=-I (negative identity matrix), the eigenvalues are -1, -1 (that is, -1 with multiplicity 2).

Eigenvectors: any vector x satisfies Ax=-x, hence any 2-dimensional vector is an eigenvector. A set of independent vectors is given by

$v^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $v^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

b) The origin is an asymptotically stable proper node.

c) The phase portrait looks like: