Quiz 2, MATH 246, Professor Radu Balan, Tuesday, 14 September 2010

(1) [4pts] A certain college graduate borrows \$8000 to buy a car. The lender charges interest at an annual rate of 10%. Assuming that interest is compounded continuously and that the borrower makes payments continuously at a constant annual rate ***k***, determine the payment rate ***k*** that is required to pay off the loan in 3 years. Also determine how much interest is paid during the 3-year period.

The following numbers might be useful: $\exp(0.3) = 1.3$, $\exp(0.3)/(\exp(0.3)-1) = 3.86$. To get full credit please make all the computations.

<u>Solution.</u>

Let S(t) denote the loan amount after *t* years. The equation governing S(t) is:

$$\frac{dS}{dt} = 0.1 \cdot S - k$$

with *S(0)=\$8000* and *k* expressed in \$/year.
The solution to this equation is:

$$S(t) = S(0)e^{rt} - \frac{k}{r}\left(e^{rt} - 1\right) = 8000e^{0.1 \cdot t} - 10k\left(e^{0.1 \cdot t} - 1\right)$$

We know that *S(3)=0*. Hence:

$$8000e^{0.3} - 10k\left(e^{0.3} - 1\right) = 0$$

$$k = \frac{800e^{0.3}}{e^{0.3} - 1} = \$800 \cdot 3.86 / year = \$8 \cdot 386 / year = \$3088 / year$$

The total interest paid the 3-year period is $3k$-S_0, that is:

$$Interest = \$3088 \cdot 3 - \$8000 = \$9264 - \$8000 = \$1264$$

(2) [6 pts] For the following differential equations: (i) determine the equilibrium points; (ii) classify each equilibrium point; and (iii) sketch the phase line portrait:

$$\text{(a)} \quad \frac{dy}{dt} = y(y-1)^2(y+2) \ , \quad -\infty < y_0 < \infty$$

$$\text{(b)} \quad \frac{dy}{dt} = e^{-y} - \mathrm{e} \ , \quad -\infty < y_0 < \infty, e = 2.71$$

(You do not have to solve these equations.)

<u>Solutions.</u>

Problem (a)

(i) The equilibrium points are: 0,1,-2.

(ii) Signature of f(y):

y		-2		0		1	
F(y)	+	0	-	0	+	0	+

Hence: -2 is (asymptotically) stable, 0 is unstable; 1 is semistable.

(iii) The phase line portrait

Problem (b)

(i) The only equilibrium point is at: y=-1.

(ii) We need the signature of f(y) around this value:

Y		-1	
f(y)	+	0	-

Hence -1 is (asymptotically) stable.

(iii) The phase line portrait: