Math 420, Spring 2023
Random Graphs: First Team Homework

Consider the datafile assigned to this homework: kn57Nodes1to57_adj20.txt
The data is formatted as follows:

```
Line 1: NumberOfNodes  NumberOfEdges
Line 2: A(1,1) A(1,2) ... A(1,n)
...
Line n+1: A(n,1) A(n,2) ... A(n,n)
```

where n=NumberOfNodes and m=NumberOfEdges.

Your task is to apply the two random graph models analyzed in class: Erdös-Renyi and SSBM. In each case you need to estimate the model parameters (p and a, b, respectively) and compare the estimated number of q-cliques given the model to the actual number of q-cliques (with $q = 3$ and $q = 4$).

1. Write a function that computes the number of q-cliques of a given graph (defined through its adjacency matrix). Make sure your function works for $q = 3$ and $q = 4$. The case $q > 4$ is optional.

2. Apply this function to your assigned data set to compute the following statistics: m, the number of edges; t, the number of 3-cliques (triangles); and f, the number of 4-cliques (complete four-vertex subgraphs).

3. For the Erdös-Renyi model $\mathcal{G}_{n,p}$ detemine the Maximum Likelihood Estimate for p that best fits the datset.

4. Compute the Expectation of the numbers of 3-cliques $X_{3,ER} = \mathcal{E}[X_3; p]$, and of 4-cliques $X_{4,ER} = \mathcal{E}[X_4; p]$ under the Erdös-Renyi model constructed at part 3.

5. For same dataset, compute the estimated parameters a, b for the Symmetric Stochastic Block Model with 2-communities $SSBM(n, 2, a, b)$ using the Method of Moments.

6. Compute the Expectation of the number of 4-cliques $X_{4,SSBM} = \mathcal{E}[X_4; a, b]$ under the $SSBM(n, 2, a, b)$ model constructed at part 5.