Math 420, Spring 2023
Geometric Graphs: Third Team Homework

Consider the text files: Target = 'Cloud_kn57Nodes1to57_coord.txt' , Source0 ='EstimatedCloud_kn57Nodes1to57_coord.txt' and nine additional files Source1 = 'NoisyCloud1_kn57Nodes1to57_coord.txt' , ..., Source9 = 'NoisyCloud9_kn57Nodes1to57_coord.txt' assigned to this homework. You can find these files into archive 'kn57Nodes1to57_coord.zip' attached to this homework. These files have the following format:

```
line 1: x1 y1 z1
line 2: x2 y2 z2
line 3: x3 y3 z3
line 4: x4 y4 z4
...
line n: xn yn zn
```

where n denotes the number of vertices (points) of a 3D geometric graph, and each line constains the coordinates of these points; the i^{th} line contains the (x, y, z) coordinates of the i^{th} point.

The files whose name contains 'Noisy' include noisy measurements of these coordinates.

Your homework is to implement a Matlab code that performs a full alignment of each of the 10 source files (Source0 to Source 9) with the target file. In each case obtain the alignment error and create a movie file that ilustrates the transformation.

I. For each pair of files (Sourcej,Target), with $j = 0, 1, 2, ..., 9$:

1. Implement the full alignment algorithm and estimate $(\hat{Q}, \hat{z}, \hat{a})$
2. Compute the alignment error $\|\hat{a}\hat{Q}(X - \hat{z}1^T) - Y\|_F$
3. Create a .avi file using the videoWriter in Matlab, saving about 101 images (plots) where the k^{th} image (k running from 0 to 100) defines the $t = k/100$ interpolation:

$$X(t) = a(t)Q(t)(X - z(t)1^T) \ , \ a(t) = 1 - t + t\hat{a} \ , \ Q(t) = J^T \cdot expm(t \cdot logm(J \cdot \hat{Q})) \ , \ z(t) = t\hat{z}$$

where $expm$ and $logm$ are Matlab matrix exponential and matrix logarithm functions, and the matrix J is diagonal and defined as follows: if $det(\hat{Q}) = +1$ then $J = I$ (i.e., the identity matrix), whereas if $det(\hat{Q}) = -1$ then its diagonal is 1 everywhere execept on one location where it is -1; try a few choices for J, e.g., $J = diag([-1, 1, 1])$ or $J = diag([1, -1, 1])$, or $J = diag([1, 1, -1])$. Each image should contain a plot of the geometric graph contained in $X(t)$. Plot the graph using the closest $K = 4$ neighbors.

II. Once the 10 pairs (Sourcej,Target) files have been processed, plot the alignment error as function of index j, where j runs from 0 to 9.

III. What type of dependency $error = error(j)$ do you you obtain? That is, is it linear, polynomial or exponential? Try also $error = error(9 - j)$. j measures the level of noise: the larger the value the larger the noise.