Math 420, Spring 2023
Second Team Homework: SIR and SEIR Models

Consider the dataset associated to your team. The problems are stated with Matlab indexing convention: from 1 to the length of the vector. The algorithms presented in class use the natural convention, where time $t_0 = 0$ may be indexed by 0. Be careful to implement algorithms correctly, regardless of indexing convention on your platform.

Exercise 1. This problem asks you to implement and run the Meta-Algorithm for SIR Calibration using the time series of *cumulative number of detected infection $V(t)$ and the* cumulative number of deaths $Y(t)$. *Use initialization $S(0) = N$, $I(0)$ from the data set, $R(0) = 0$. Use the following parameters: $T_{max} = 119$ (i.e., a total of 120 time samples), $V_{min} = 5$, $\tau_0 = 7$, $p = 1, 2, \infty$ (check all three values), $\lambda = 1$. For the set Ω write nested loops over α, R_0, and N by searching over the following intervals:*

$$\alpha \in [0.05, 0.2]\ ,\ R_0 \in [1.5, 1.9]\ ,\ \frac{N}{Population} \in [2\%, 10\%]$$

Recall $\beta = R_0\alpha$, and $Population$ is extracted from your data set.

(1) *Find and print the minimum of the objective function J. Print the optimal values of $\hat{\alpha}, \hat{\beta}, \hat{R}_0, \hat{N}$ and $\hat{\gamma}$ ($\hat{\gamma}$ is obtained by implementing the minimization of $\|Y - \gamma R_{sim}\|_p$ over γ).*

(2) *Visualize the $2D$ surface (function) $(\alpha, \beta) \mapsto J = J(\alpha, \beta, \hat{N}, \hat{\gamma})$ at the optimal values $\hat{N}$ and $\hat{\gamma}$ obtained before. You should obtain one plot for each p.*

(3) *Plot on the same graph the simulated I_{sim} and the preprocessed rate of detected infections $I(t) = V(t + t_0 + \tau_0) - V(t + t_0 - \tau_0)$. Plot on the same graph the predicted number of deaths $Y_{sim} = \hat{\gamma} R_{sim}$ and the observed number of deaths $Y(t)$ from your data set. You should obtain two plots for each value of p.*

(4) *If you are asked to improve the estimates, what would be your refined range of parameters?*

NOTE: the Euler scheme produces an estimate of the solution every $h = 1/100$ day time step. However the experimental data is measured **daily**. *Hence you need to downsample the output of the Euler scheme by $1/h = 100$.*

Exercise 2. *This problem asks you to implement and run the Meta-Algorithm for SEIR Calibration using the time series of* cumulative number of detected infection $V(t)$ and the *cumulative number of deaths $Y(t)$.*

Use the same parameters as in Problem 1 with the following initialization: $S(0) = N$, $E(0) = I(0)$ to match the data set value $V(t_0 + \tau_0) - V(t_0 - \tau_0)$, $R(0) = 0$. For the set Ω write nested loops over α, R_0, and N by searching over the following intervals:

$$\alpha \in [0.05, 0.4]\ ,\ \delta \in [0.05, 0.4]\ ,\ R_0 \in [1.5, 1.9]\ ,\ \frac{N}{Population} \in [2\%, 10\%]$$

Recall $\beta = R_0\alpha$, and $Population$ is extracted from your data set.

(1) Find and print the minimum of the objective function J. Print the optimal values of $\hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{R}_0, \hat{N}$ and $\hat{\gamma}$ ($\hat{\gamma}$ is obtained by implementing the minimization of $\|Y - \gamma R_{sim}\|_p$ over γ).

(2) Visualize the $2D$ surfaces (functions):

- $(\alpha, \beta) \mapsto J = J(\alpha, \beta, \hat{\delta}, \hat{N}, \hat{\gamma})$ at the optimal values $\hat{\delta}$, $\hat{N}$ and $\hat{\gamma}$ obtained before. You should obtain one plot for each p.
- $(\alpha, \delta) \mapsto J = J(\alpha, \hat{\beta}, \delta, \hat{N}, \hat{\gamma})$ at the optimal values $\hat{\beta}$, $\hat{N}$ and $\hat{\gamma}$ obtained before. You should obtain one plot for each p.
- $(\beta, \delta) \mapsto J = J(\hat{\alpha}, \beta, \delta, \hat{N}, \hat{\gamma})$ at the optimal values $\hat{\alpha}$, $\hat{N}$ and $\hat{\gamma}$ obtained before. You should obtain one plot for each p.

(3) Plot on the same graph the simulated I_{sim} and the preprocessed rate of detected infections $I(t) = V(t+t_0+\tau_0) - V(t+t_0-\tau_0)$. Plot on the same graph the predicted number of deaths $Y_{sim} = \hat{\gamma} R_{sim}$ and the observed number of deaths $Y(t)$ from your data set. You should obtain two plots for each value of p.

(4) If you are asked to improve the estimates, what would be your refined range of parameters?