AMSC/MATH 420, Spring 2024
First Solo Homework:
Linear Algebra and Geometry

For these problems you do not need a calculator.

Problem I

For each matrix, indicate if they are symmetric, orthogonal, or positive semidefinite?

1.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 0 \end{bmatrix}$$

2.

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

3.

$$C = 1/3 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

4.

$$D = 1/\sqrt{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

Problem II

The 4x4 symmetric matrix $A \in \mathbb{R}^{4\times 4}$ diagonalizes as follows

$$A = \begin{bmatrix} 1 & 2 & -0.5 & 1.5 \\ 2 & 1 & 1.5 & -0.5 \\ -0.5 & 1.5 & 1 & 2 \\ 1.5 & -0.5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \end{bmatrix}$$

Compute the following:

1.

$$\begin{aligned} a \;=\; & \text{maximum} \quad x^T A x \\ & \text{subject to} \\ & x^T x = 1 \end{aligned}$$

2.

$$\begin{aligned} b \;=\; & \text{minimum} \quad x^T A x \\ & \text{subject to} \\ & x^T x \leq 1 \end{aligned}$$

3.

$$
\begin{aligned}
b \; = \quad & \text{maximum} \qquad \frac{x^T A x}{x^T x} \\
& \text{subject to} \\
& x \neq 0
\end{aligned}
$$

Problem III

Consider the two-dimensional surface in $\mathbb{R}^3$ defined by the equation

$$6x^2 + 8xy + 4xz + 13y^2 + 6yz + z^2 = 38$$

1. Prove that the point $P = (1, 1, 1)$ belongs to this surface;
2. Determine the normal line to this surface, passing through P;
3. Find two independent vectors that are tangent to this surface at P. Can you find three independent vectors tangent to the surface?

Total: 10 points