Math 420, Spring 2024
Second Team Homework: SIR and SEIR Models

Consider the dataset associated to your team. The problems are stated with Matlab indexing convention: from 1 to the length of the vector. The algorithms presented in class use the natural convention, where time $t_0 = 0$ may be indexed by 0. Be careful to implement algorithms correctly, regardless of indexing convention on your platform.

Exercise 1. This problem asks you to implement and run the Meta-Algorithm for SIR Calibration using the time series of *cumulative number of detected infection $V(t)$ and the* cumulative number of deaths $Y(t)$. *Use initialization $S(0) = N$, $I(0)$ from the data set, $R(0) = 0$. Use the following parameters: $T_{max} = 119$ (i.e., a total of 120 time samples), $V_{min} = 5$, $\tau_0 = 7$, $p = 1, 2, \infty$ (check all three values), $(c_I, c_Y) = (0, 1)$ and $(C_I, c_Y) = (1, 1)$ (check both values). For the set Ω write nested loops over α and R_0 by searching over the following intervals:*

$$\alpha \in [0.05, 0.4] \; stepsize = 0.01 \; , \; R_0 \in [0.8, 2.2] \; stepsize = 0.05$$

Recall $\beta = R_0\alpha$, and $N = Population$ is extracted from your data set.

(1) *Find and print the minimum of the objective function J. Print the optimal values of $\hat{\alpha}, \hat{\beta}, \hat{R}_0$ and $\hat{\gamma}, r\hat{h}o$ ($\hat{\gamma}$ is obtained by implementing the minimization of $\|Y - \gamma R_{sim}\|_p$ over γ, whereas $r\hat{h}o$ is obtained by minimization of $\|I - \rho I_{sim}\|_p$ over ρ).*

(2) *Visualize the 2D surface (function) $(\alpha, \beta) \mapsto J = J(\alpha, \beta; \hat{\gamma}, \hat{\rho})$ at the optimal values $\hat{\gamma}$ and $r\hat{h}o$ for each value of (α, β). You should obtain one plot for each p.*

(3) *For $(\alpha, \beta) = (\hat{\alpha}, \hat{\beta})$, plot on the same graph the simulated I_{sim} and the preprocessed rate of detected infections $I(t) = V(t+t_0+\tau_0) - V(t+t_0-\tau_0)$. Plot on the same graph the predicted number of deaths $Y_{sim} = \hat{\gamma} R_{sim}$ and the observed number of deaths $Y(t)$ from your data set (make sure to align Y with the same first time t_0) . You should obtain two plots for each value of p.*

(4) *Repeat the previous questions for each pair (c_I, c_Y).*

NOTE: the Euler scheme produces an estimate of the solution every $h = 1/100$ day time step. However the experimental data is measured **daily**. *Hence you need to downsample the output of the Euler scheme by $1/h = 100$.*

Exercise 2. *This problem asks you to implement and run the Meta-Algorithm for SEIR Calibration using the time series of* cumulative number of detected infection $V(t)$ and the *cumulative number of deaths $Y(t)$.*

Use the same parameters as in Problem 1 with the following initialization: $S(0) = N$, $E(0) = I(0)$ to match the data set value $V(t_0 + \tau_0) - V(t_0 - \tau_0)$, $R(0) = 0$. Use the following parameters: $T_{max} = 119$ (i.e., a total of 120 time samples), $V_{min} = 5$, $\tau_0 = 7$, $p = 1, 2, \infty$ (check all three values), $(c_I, c_Y) = (0, 1)$ and $(C_I, c_Y) = (1, 1)$ (check both values). For the set Ω write nested loops over α, R_0, and δ by searching over the following intervals:

$$\alpha \in [0.05, 0.4] \; stepsize = 0.01 \; , \; \delta \in [0.05, 0.4] \; stepsize = 0.01, \; R_0 \in [0.8, 2.2] \; stepsize = 0.05$$

Recall $\beta = R_0\alpha$, and $N = Population$ is extracted from your data set.

(1) Find and print the minimum of the objective function J. Print the optimal values of $\hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{R}_0$ and $\hat{\gamma}, \hat{\rho}$ ($\hat{\gamma}$ is obtained by implementing the minimization of $\|Y - \gamma R_{sim}\|_p$ over γ, whereas $r\hat{h}o$ is obtained by minimization of $\|I - \rho I_{sim}\|_p$ over ρ).

(2) Visualize the $2D$ surfaces (functions):
- $(\alpha, \beta) \mapsto J = J(\alpha, \beta, \hat{\delta}; \hat{\gamma}, \hat{\rho})$ at the optimal values $\hat{\delta}$, $\hat{\gamma}$, and $\hat{\rho}$ obtained before. You should obtain one plot for each p.
- $(\alpha, \delta) \mapsto J = J(\alpha, \hat{\beta}, \delta; \hat{\gamma}, \hat{\rho})$ at the optimal values $\hat{\beta}$, $\hat{\gamma}$ and $\hat{\rho}$ obtained before. You should obtain one plot for each p.
- $(\beta, \delta) \mapsto J = J(\hat{\alpha}, \beta, \delta; \hat{\gamma}, \hat{\rho})$ at the optimal values $\hat{\alpha}$, $\hat{\gamma}$ and $\hat{\rho}$ obtained before. You should obtain one plot for each p.

(3) For $(\alpha, \beta, \delta) = (\hat{\alpha}, \hat{\beta}, \hat{\delta})$, plot on the same graph the simulated I_{sim} and the preprocessed rate of detected infections $I(t) = V(t + t_0 + \tau_0) - V(t + t_0 - \tau_0)$. Plot on the same graph the predicted number of deaths $Y_{sim} = \hat{\gamma} R_{sim}$ and the observed number of deaths $Y(t)$ from your data set. You should obtain two plots for each value of p.

(4) Repeat the previous questions for each pair (c_I, c_Y).