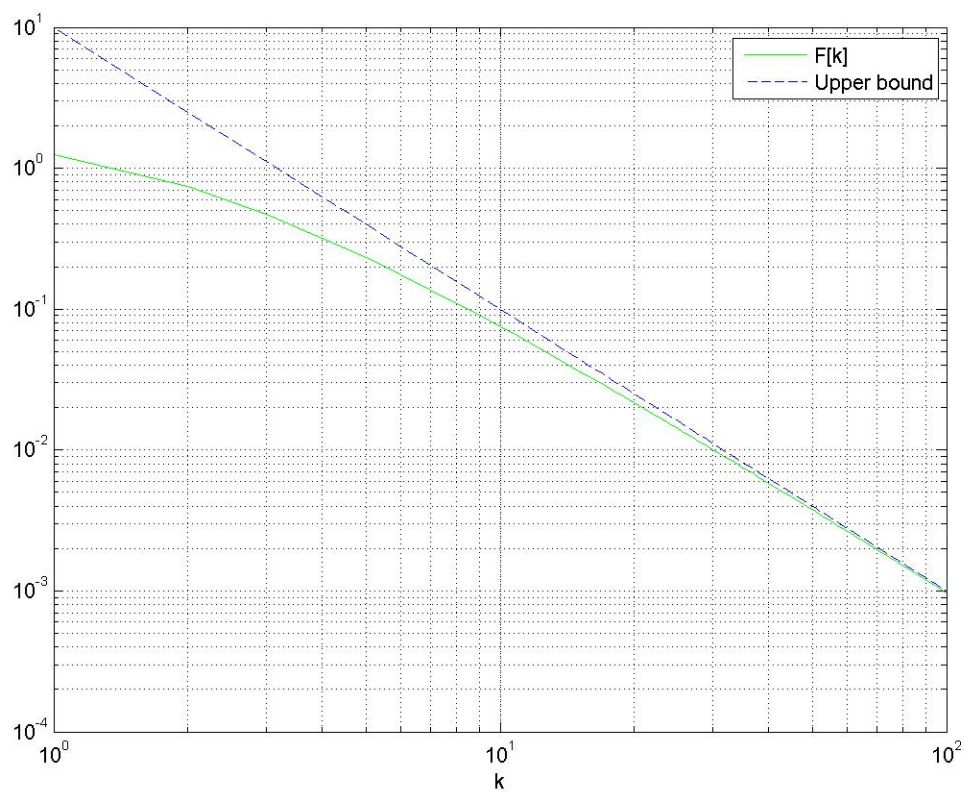
Homework #4
Due: Tuesday, September 25, 2012

Note: Use of Matlab (or any other software) is not permitted.

I. (2pts) An even function f:[-1/2,1/2)→R has the Fourier coefficients F[k] decaying to 0 as depicted in the figure below using green color(note the plot is in log-log scale). An upper bound on the coefficients is depicted using blue dotted color as a straight line in the log-log scale. (The blue line is guaranteed to stay above the green curve for all values of k larger than 1.) Estimate the rank N of the partial Fourier sum $\sum_{k=-N}^{k} F[k]e^{2\pi ikx}$ that achieves an error no larger than 0.01 at every point in [-1/2,1/2).
Hints: 1) estimate the slope and y-crossing of the blue curve; 2) use this information to construct an upper bound on F[k]; 3) use the approximation formula from previous homework.



II. (2pts) Consider an odd function f:[-1/2,1/2))→R whose Fourier coefficients F[k] are bounded by $|F[k]| \leq 9 \cdot 10^{-|k|}$. How many terms N are needed in the partial Fourier sum $\sum_{k=-N}^{k} F[k]e^{2\pi ikx}$ to has an error no larger than 0.002 at every point?

III. Compute the Fourier transform of the following functions (1-6):

1. $f(x) = \begin{cases} 1 & , 1 \le x \le 2 \\ 0 & , otherwise \end{cases}$

2. $f(x) = e^{-a|x|}$, for some a>0.

3. $f(x) = \dfrac{1}{x^2 + a^2}$, for some a>0. (Hint: Use synthesis formula and exercise III.2)

4. $f(x) = \begin{cases} \sin(2\pi x) & , 1 \le x \le 2 \\ 0 & , otherwise \end{cases}$

5. $f(x) = e^{-|x|} \cos(2\pi a x)$, for some a>0.

6. $f(x) = \dfrac{\cos(2\pi x)}{x^2 + a^2}$, for some a>0.

Total: 10 pts (2 points I&II, 1 point each in III)