1.

Method 1

Let $g(x) = e^{-2\pi i 7000x} f(x)$. Note $G(s) = F(s+7000)$ and thus $G(s) = 0$ for $|s| > 5000$. Hence $g \in B_{5000}$ (that is, g is $5000Hz$-band limited). Shannon's sampling formula yields

$$g(x) = \sum_n g(nT) sinc(\frac{x-nT}{T}) \quad , \quad T = \frac{1}{10000} = 100\mu s$$

But $g(nT) = e^{-\frac{7}{5}\pi i n} f(nT) =$. Thus

$$f(x) = e^{14000\pi i x} g(x) = \sum_n f(nT) e^{-\frac{7}{5}\pi i n} e^{14000\pi i x} sinc(\frac{x-nT}{T})$$

Method 2

Use:

$$f(x) = \int_{-\infty}^{\infty} F(s) e^{2\pi i s x} ds = \int_{2000}^{12000} e^{2\pi i s x} F(s) ds$$

But

$$F(s) = \sum_n c_n e^{2\pi i n s/(10000)} = \sum_n c_n e^{2\pi i n T s}$$

Now

$$c_n = \frac{1}{10000} \int_{2000}^{12000} F(s) e^{-2\pi i n s/10000} ds = T f(-nT)$$

Thus

$$f(x) = \sum_n T f(-nT) \int_{2000}^{12000} e^{2\pi i s x + 2\pi i s n T} ds = \sum_n T f(nT) \int_{2000}^{12000} e^{2\pi i s(x-nT)} ds =$$

$$= \sum_n T f(nT) \frac{1}{2\pi i (x-nT)} \left(e^{2\pi i 12000(x-nT)} - e^{2\pi i 2000(x-nT)} \right) = \sum_n f(nT) \frac{e^{2\pi i 7000(x-nT)}}{2\pi i \frac{x-nT}{T}} \left(e^{\pi i \frac{x-nT}{T}} - e^{-\pi i \frac{x-nT}{T}} \right) =$$

$$= \sum_n f(nT) e^{14000\pi i x - 14\pi i n/10} sinc\left(\frac{x-nT}{T}\right)$$

2.

The maximum sampling period is $T = \frac{1}{2\cdot 50000} = \frac{1}{100000}s = 10^{-5}s = 10\mu s$.

3.

The sampling period is $T = \frac{1}{2\cdot 10000}s = 50\mu s$ (Nyquist rate). Hence the Shannon's formula becomes:

$$f(x) = \sum_n f(nT) sinc(\frac{x-nT}{T} = f(-24T) sinc(\frac{x+1.2\cdot 10^{-3}}{5\cdot 10^{-5}}) + f(4T) sinc(\frac{x-0.2\cdot 10^{-3}}{5\cdot 10^{-5}}) = -2 sinc(20000x+24) + si$$

Thus

$$f(0) = 0$$

and

$$f(10^{-6}) = -2 sinc(0.02+24) + sinc(0.02-4) = -2 sinc(24.02) - sinc(3.98)$$

4.

The maximal reconstruction error is given by

$$|f(x) - \sum_n f(nT) sinc(\frac{x-nT}{T})| \leq 2 \int_{|\omega| \geq \frac{1}{2T}}^{\infty} e^{-|\omega|} d\omega = 4 \int_{500}^{\infty} e^{-\omega} d\omega = 4e^{-500}$$