MATH 464 – Test 2
November 13, 2012

I. (10pts) Find all generalized functions *f* that satisfy the following inhomogeneous equation:

$$(x^4+1)\cdot f(x)=\delta(x).$$

II. (10pts) Consider the polynomial $A(x)=(x-1)^2(x+2)(x^2+x+1)$. Find all generalized functions *f* that satisfy the homogeneous equation:

$$A(x)\cdot f(x)=0.$$

III. (20pts) Find the Fourier transform of each of the following generalized functions. In so doing, freely use the rules from the Fourier transform calculus together with the table of Fourier transforms.

a. (10pts) $f(x)=e^{4\pi ix}\cdot sign(x)$;

b. (10pts) $f(x)=\dfrac{1}{x^2-4}$.

IV. (30pts) Consider the functions *f* defined by

$$f(x)=\begin{cases}-2 & if \quad x<-\frac{\pi}{2}\\ 0 & if \quad -\frac{\pi}{2}\le x<0\\ x & if \quad 0\le x\end{cases}.$$

a. (10pts) Draw the plot of *f* on the range [-3,3].
b. (10pts) Compute the derivative *f'* of *f* in the sense of distributions.
c. (10pts) Consider the test function $\Phi(x)=e^{-\pi x^2}$. Compute $f'\{\Phi\}$.

V. (10pts) What is the maximum sampling period T_s of a 10KHz-bandlimited signal *f* so that we can recover *f(t)* from its samples $\{f(nT_s),\ n\in Z\}$?

VI. (20pts) A signal *f* is supported in frequency domain in the frequency band [-25KHz, 75KHz].

a. (10pts) What is the maximum sampling period T_s so that we can recover *f(t)* from its samples $\{f(nT_s),\ n\in Z\}$?
b. (10pts) What is the best approximation of *f(t)* from samples $\{f(nT),\ n\in Z\}$ obtained at sampling period $T=10\mu s$? Can you recover *f(t)* for all *t* with no reconstruction error?

(Recall $1KHz = 1000Hz$, $1\mu s = 10^{-6}\ s$)

Total: 100 pts