I. Since $x^4+1=0$ has no real solutions, the only generalized function that satisfies the inhomogeneous equation is given by

$$f(x) = \frac{\delta(x)}{x^4+1} = \delta(x).$$

II. The general solution is given by

$$f(x) = c_1\delta(x-1) + c_2\delta'(x-1) + c_3\delta(x+2).$$

III.
(a) Recall $\widehat{p_{-1}}(s) = -i\pi sign(s)$ and $\widehat{sign}(s) = \frac{1}{i\pi}p_{-1}(s)$ (see the tables at the end of the textbook). Thus

$$\hat{f}(s) = \widehat{sign}(s-2) = -\frac{i}{\pi}p_{-1}(s-2).$$

(b) Note $f(x) = \frac{1}{4}\frac{1}{x-2} - \frac{1}{4}\frac{1}{x+2} = \frac{1}{4}p_{-1}(x-2) - \frac{1}{4}p_{-1}(x+2)$. Thus

$$\hat{f}(s) = \frac{1}{4}e^{-4\pi is}\widehat{p_{-1}}(s) - \frac{1}{4}e^{4\pi is}\widehat{p_{-1}}(s) = -\frac{i\pi}{4}e^{-4\pi is}sign(s) + \frac{i\pi}{4}e^{4\pi is}sign(s) =$$

$$= -\frac{\pi}{2}sin(4\pi s)sign(s) = -\frac{\pi}{2}sin(4\pi|s|).$$

IV.
(a) _ – /
(b) The derivative is

$$f'(x) = 2\delta(x+\frac{\pi}{2}) + h(x)$$

where $h(x)$ is the Heaviside step function.
(c) Thus

$$f'\{\Phi\} = 2e^{-\pi(-\pi/2)^2} + \int_0^\infty e^{-\pi x^2}dx = 2e^{-\pi^3/4} + \frac{1}{2}\int_{-\infty}^\infty e^{-\pi x^2}dx = 2e^{-\pi^3/4} + \frac{1}{2}.$$

V. The maximum sampling period is

$$T_s = \frac{1}{2\cdot 10000} = \frac{1}{20000}s = 5\cdot 10^{-5}s = 50\mu s.$$

VI.
(a) The maximum sampling period is

$$T_s = \frac{1}{(75-(-25))\cdot 10^3}s = 1\cdot 10^{-5}s = 10\mu s.$$

(b) Th exact reconstruction (i.e. no error) is possible for $T = 10\mu s$. This is given by

$$f(t) = \sum_{n=-\infty}^{\infty} f(nT)e^{2\pi i 25000(t-nT)}sinc\left(\frac{t-nT}{T}\right).$$