Homework #3
Due: Tuesday, February 15, 2011

Compute the Fourier coefficients, and expand in Fourier series the following 1-periodic functions.
Note: Use of Matlab (or any other software) is not permitted.

1. $\frac{\pi}{2}(1-2x)$, for $0 < x < 1$

2. x^2 , for $0 < x < 1$

3. x^3 , for $0 < x < 1$

4. i) Compute the Fourier coefficients, and expand in Fourier series the following 1-periodic function:

$$f(x) = \begin{cases} x & \text{for } 0 < x < \frac{1}{2} \\ a & \text{for } x = \frac{1}{2} \\ 0 & \text{for } \frac{1}{2} < x < 1 \end{cases}$$

ii) What should a be so that the Fourier series converges to $f(x)$ for every $0<x<1$?

5. i) Compute the Fourier coefficients, and expand in Fourier series the following 1-periodic function.

$$f(x) = \begin{cases} 4x & 0 < x \le \frac{1}{4} \\ 2-4x & \frac{1}{4} < x \le \frac{1}{2} \\ 4x-2 & \frac{1}{2} < x \le \frac{3}{4} \\ 4-4x & \frac{3}{4} < x < 1 \end{cases}$$

ii) Is the Fourier series convergent for every $0<x<1$?

In the following three problems the functions are extended by periodicity outside the interval of definition. Compute the Fourier coefficients, and expand in Fourier series the following 1-periodic functions.

6. $\sin(\pi x)$, for $-\frac{1}{2} < x < \frac{1}{2}$

7. $\cos(\pi x)$, for $-\frac{1}{2} < x < \frac{1}{2}$

8. $e^{\pi i x}$, for $-\frac{1}{2} < x < \frac{1}{2}$

9. Consider a 1-periodic C^1 function $f: [0,1) \to R$ whose Fourier coefficients F[n] decay to 0 as $|n| \to \infty$ bounded above by $|F[n]| \leq \frac{10}{|n|}$, for $|n| \geq 1$. Estimate the minimum degree N of a symmetric trigonometric polynomial $A_N(x) = \sum_{k=-N}^{N} c_k e^{2\pi i x}$ that approximates with a 5% mean square error.

10. Consider a 1-periodic function $f: [0,1) \to R$ whose Fourier coefficients F[n] decay to 0 as $|n| \to \infty$ bounded above by $|F[n]| \leq \frac{10}{n^4}$, for $|n| \geq 1$. Estimate the minimum degree N of a symmetric trigonometric polynomial $A_N(x) = \sum_{k=-N}^{N} c_k e^{2\pi i x}$ that approximates with a 1% mean square error.
Note: You may use the following inequality: $\sum_{n=N+1}^{\infty} \frac{1}{n^8} \leq \frac{1}{7N^7}$.

Total: 10 pts (1 point each problem)