

Homework #4

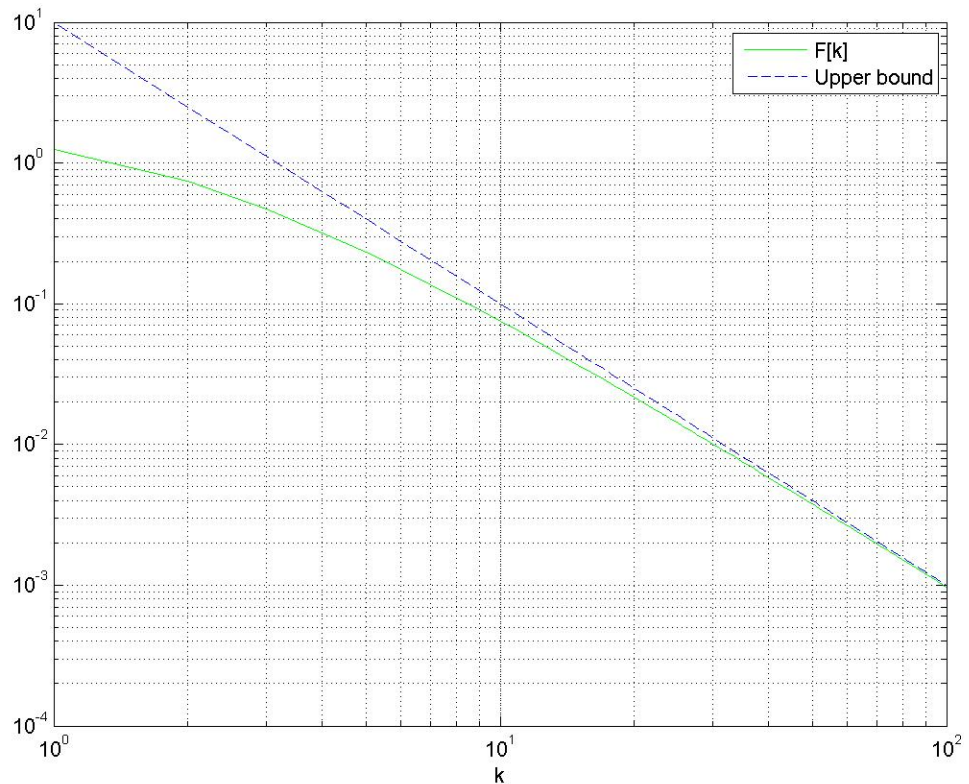
Due: Tuesday, February 22, 2011

Note: Use of Matlab (or any other software) is not permitted.

- I.** (2pts) An even function $f: [-1/2, 1/2] \rightarrow \mathbb{R}$ has the Fourier coefficients $F[k]$ decaying to 0 as depicted in the figure below using green color (note the plot is in log-log scale). An upper bound on the coefficients is depicted using blue dotted color as a straight line in the log-log scale. (The blue line is guaranteed to stay above the green curve for all values of k larger than 1.) Estimate the rank N of the partial Fourier sum

$$\sum_{k=-N}^k F[k] e^{2\pi i k x} \text{ that achieves an accuracy of 1\% at every point in } [-1/2, 1/2].$$

Hints: 1) estimate the slope and y-crossing of the blue curve; 2) use this information to construct an upper bound on $F[k]$; 3) use the approximation formula from previous homework.



- II.** (2pts) Consider an odd function $f: [-1/2, 1/2] \rightarrow \mathbb{R}$ whose Fourier coefficients $F[k]$ are bounded by $|F[k]| \leq 9 \cdot 10^{-|k|}$. How many terms N are needed in the partial Fourier sum $\sum_{k=-N}^k F[k] e^{2\pi i k x}$ to achieve an accuracy of 0.2% at every point?

Hint: Use the sum of a geometric series you have been provided in homework 1.

III. Compute the Fourier transform of the following functions (1-6):

1.
$$f(x) = \begin{cases} 1 & , 1 \leq x \leq 2 \\ 0 & , \text{otherwise} \end{cases}$$

2.
$$f(x) = e^{-a|x|} , \text{ for some } a > 0.$$

3.
$$f(x) = \frac{1}{x^2 + a^2} , \text{ for some } a > 0. \text{ (Hint: Use synthesis formula and exercise III.2)}$$

4.
$$f(x) = \begin{cases} \sin(2\pi x) & , 1 \leq x \leq 2 \\ 0 & , \text{otherwise} \end{cases}$$

5.
$$f(x) = e^{-|x|} \cos(2\pi ax) , \text{ for some } a > 0.$$

6.
$$f(x) = \frac{\cos(2\pi x)}{x^2 + a^2} , \text{ for some } a > 0.$$

Total: 10 pts (2 points I&II, 1 point each in III)