Homework #6
Due: Thursday, March 10, 2011

1. (2pts) Find the Fourier transform of the function $f:R\to R$

$$f(x) = \int_{-1/2}^{1/2} e^{-\pi(x-u)^2}\,du$$

2. (2pts) Let $f_0, f_1 : R\to R$ be functions defined by

$$f_0(x) = e^{-x^2} \quad , \quad f_1(x) = xe^{-x^2}$$

Compute the following convolutions:

i) $f_0 * f_0$

ii) $f_0 * f_1$.

3. (3pts) Find a non-zero function f on R that satisfies the following equation:

$$\int_{-\infty}^{\infty} f(u)\,f(x-u)\,du = f(x) \ , \ -\infty < x < \infty$$

4. (3pts) Find a continuous function $f:R\to R$ that satisfies

$$-f'(x) + f(x) = \begin{cases} e^{-x} & , \ x > 0 \\ 0 & , \ \mathrm{x} < 0 \end{cases}$$

Total: 10pts.