

Homework #9
Due: Thursday, April 21, 2011

Note: Use of Matlab (or any other software) is not permitted.

I. (Exercise 7.33) Compute the Fourier transform of the following generalized functions.

1. $f(x) = x \cdot \delta'(x)$
2. $f(x) = \cos^2(\pi x)$
3. $f(x) = e^{-\pi x^2} * 2\pi x$

II. (Exercises 7.14) Let a, b, c, d be real numbers with $a > 0$ and $c > 0$. Find a simple representation for the convolution product $\beta * f$ when:

4. (1pt) $\beta(x) = \delta(ax + b), f(x) = \cos(cx + d);$
5. (1pt) $\beta(x) = e^{-ax^2}, f(x) = \cos(cx + d);$
6. (1pt) $\beta(x) = \delta'(ax + b), f(x) = \operatorname{sgn}(x).$

III. (Exercise 7.26) Find all generalized functions f that satisfy each of the following homogeneous equations.

7. (1pt) $(x^2 - 1) \cdot f(x) = 0;$
8. (1pt) $(x^4 - 1) \cdot f(x) = 0;$
9. (1pt) $(x^4 - 1)^2 \cdot f(x) = 0$

IV. Compute the derivative in distribution (or weak) sense of the following function:

10. (1pt)
$$f(x) = \begin{cases} x^2 - 2 & \text{if } x < 0 \\ \sin(x) & \text{if } 0 \leq x < \frac{\pi}{2} \\ 2 \cos(x) + x & \text{if } \frac{\pi}{2} \leq x \end{cases}$$

Total: 10 pts (1 point each)