1.

$$\frac{\pi}{2}(1-2x) = \sum_{k=1}^{\infty} \frac{\sin(2\pi kx)}{k}$$

2.

$$x^2 = \frac{1}{3} - \frac{1}{\pi}\sum_{k=1}^{\infty}\frac{\sin(2\pi kx)}{k} + \frac{1}{\pi^2}\sum_{k=1}^{\infty}\frac{\cos(2\pi kx)}{k^2}$$

3.

$$x^3 = \frac{1}{4} + \sum_{k=1}^{\infty}\left[\left(\frac{3}{2\pi^3 k^3} - \frac{1}{\pi k}\right)\sin(2\pi kx) + \frac{3}{2\pi^2 k^2}\cos(2\pi kx)\right]$$

4.

$$f(x) = \frac{1}{8} + \sum_{k\neq 0}\left(\frac{i(-1)^k}{4\pi k} + \frac{(-1)^k - 1}{4\pi^2 k^2}\right)e^{2\pi ikx} = \frac{1}{8} - \frac{1}{2\pi}\sum_{k>0}\frac{(-1)^k}{k}sin(2\pi kx) - \frac{1}{\pi^2}\sum_{p>0}\frac{1}{(2p+1)^2}cos(2\pi(2p+1)x)$$

To have pointwise convergence, $a = \frac{f(0.5-)+f(0.5)}{2} = \frac{1}{4}$.

5.

$$f(x) = \frac{1}{2} - \frac{4}{\pi^2}\sum_{n\geq 0}\frac{1}{(2n+1)^2}\cos(4\pi(2n+1)x)$$

The Fourier series converges pointwise at every $x$.

6.

$$\begin{aligned}\sin(\pi x) &= \frac{2}{\pi}\left(\frac{1}{1^2-1/4}\sin(2\pi x) - \frac{2}{2^2-1/4}\sin(4\pi x) + \cdots\right)\\ &= \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{k(-1)^{k+1}}{k^2-1/4}\sin(2\pi kx)\end{aligned}$$

7.

$$\begin{aligned}\cos(\pi x) &= \frac{2}{\pi}\left(1 + \frac{1}{2(1^2-1/4)}\cos(2\pi x) - \frac{1}{2(2^2-1/4)}\cos(4\pi x) + \cdots\right)\\ &= \frac{2}{\pi}\left(1 + \sum_{k=1}^{\infty}\frac{1}{2(k^2-1/4)}\cos(2\pi kx)\right)\end{aligned}$$

8.

$$e^{\pi ix} = \sum_{k=-\infty}^{\infty}\frac{(-1)^{k+1}}{\pi(k-1/2)}e^{2\pi ikx}$$

9. Use the following estimate for MMSE:

$$MMSE = \sum_{|n|>N}|F[n]|^2 \leq \sum_{|n|>N}\frac{100}{n^2} \leq 2\frac{100}{N} = \frac{200}{N}$$

An estimate for N that guarantees MSE less than 5% is obtained by imposing

$$\frac{200}{N} \leq \frac{5}{100} \rightarrow N \geq 4000$$

The minimum degree $N$ is thus 4000.

10. Use the following estimate for MMSE:

$$MMSE = \sum_{|n|>N} |F[n]|^2 \leq \sum_{|n|>N} \frac{100}{n^8} \leq 2\frac{100}{7N^7} = \frac{200}{7N^7}$$

An estimate for N that guarantees MSE less than 1% is obtained by imposing

$$\frac{200}{7N^7} \leq \frac{1}{100} \rightarrow N \geq \left(\frac{20000}{7}\right)^{\frac{1}{7}} \approx 3.11$$

Hence the minimum degree $N$ is 4.

(You can leave the result as $(20000/7)^{1/7}$.)