1. (10pt) Let \( x(t) \) be a solution of \( \dot{x} = A(t)x + h(t) \), where \( A(t) \) and \( h(t) \) are continuous on an open interval \( 0 < t < \infty \). Prove that \( |x(t)| \) is bounded for \( t \geq 1 \), if both \( \int_1^\infty |A(t)| \, dt < \infty \) and \( \int_1^\infty |h(t)| \, dt < \infty \).

2. (10 pt) Let \( A : I \to M_d(R) \) and \( B : I \to M_d(R) \) be differentiable functions on the interval \( I \), that is, every entry of \( A(t) \) and \( B(t) \) is just a real-valued differentiable function on an interval. Prove that
\[
\frac{d}{dt} [A(t)B(t)] = \dot{A}(t)B(t) + A(t)\dot{B}(t)
\]

3. (10pt) Consider \( \dot{x} = A(t)x + h(t) \), where
\[
A = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix} \quad \text{and} \quad h(t) = \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}.
\]
Verify that
\[
X(t) = \begin{bmatrix} e^{3t} & te^{3t} \\ 0 & e^{3t} \end{bmatrix}
\]
is a fundamental matrix solution of \( \dot{x} = A(t)x + h(t) \). Find a solution to the initial value problem
\[
\dot{x} = A(t)x + h(t) \quad \text{and} \quad x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.
\]

4. (10pt) Suppose \( A : (0, \infty) \to M_d(R) \) is continuous. Prove the following: If
\[
\int_1^\infty Tr A(t) \, dt = \infty
\]
Then there exists a solution \( x(t) \) of \( \dot{x} = A(t)x \) such that \( |x(t)| \) is unbounded for \( t \geq 1 \).

Total: 40 pts