Reliability Inference in a First Hitting Time Model with Augmented Data

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OUTLINE

I. Reliability Data based on Marker and Degradation Processes

II. Failure Models based on (Bivariate) Wiener Processes

III. Derivation of Likelihood Based on Reflection Principle

IV. Information Comparisons versus Marker-only Data

V. Inference & Prediction – simulations and illustration

VI. Summary & Conclusions

Failure Modes and Markers

Study probability distribution of Time to Failure, where Failure is interpreted (or **defined**) as time S for Degradation Process $X(t)$ to cross a Threshold a .

Degradation may be *latent* (unobservable) or else prohibitively difficult or expensive to measure.

So model in terms of Marker variables $Y(t)$ easier to measure.

Example, in ball bearing fans.
- degradation: surface defect or roughness of bearing balls.
- marker: Vibration (Hz).

Failure Mechanisms in Electronics

Failures in electronics can be result of one or more failure mechanisms

- Overstress mechanisms (stress exceeds item strength; failure is sudden)
- Wearout mechanisms (Accumulation of damage with repeated stress)

Examples of wearout failure mechanisms

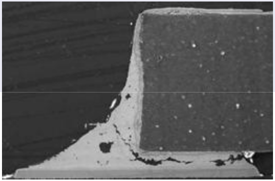
- Mechanical - Fatigue, Creep, Wear
- Electrical - Electromigration
- Chemical - Corrosion, dendrite growth, intermetallic growth

Degradation Process

Interested in the evolution of latent degradation processes: underlying unobserved processes that act on an item and eventually cause it to fail

Examples of latent degradation variables in electronics

- length of a crack in a solder joint
- corrosion level of solder joints
- random effect



Time-to-failure *

is determined by the first time the degradation variable first crosses to a critical level or *threshold* (random or fixed)

Failure Mechanism	**Failure Sites**	**Failure Causes**	**Failure Models**
Fatigue	Die attach, Wirebond/TAB, Solder leads, Bond pads, Traces, Vias/PTHs, Interfaces	Cyclic Deformations (Δ T, Δ H, Δ V)	Nonlinear Power Law (Coffin-Manson)
Corrosion	Metallizations	M, ΔV, T, chemical	Eyring (Howard)
Electromigration	Metallizations	T, J	Eyring (Black)
Conductive Filament Formation	Between Metallizations	M, ΛV	Power Law (Rudra)
Stress Driven Diffusion Voiding	Metal Traces	σ, T	Eyring (Okabayashi)
Time Dependent Dielectric Breakdown	Dielectric layers	V, T	Arrhenius (Fowler-Nordheim)

Degradation Variables

- Examples of degradation variables:
  - Length of a crack, corrosion level, surface roughness of bearing balls, light intensity of light emitting diodes
- Degradation variables are often not observable (latent)
  - In systems with complex electronics, no one variable is known to represent degradation
  - In electronic components, a degradation variable may not be measurable
- **When degradation is latent**, predictions must be based on *marker (surrogate) variables*

Marker Variables

- A marker is a random variable, which
  - Covaries with degradation and assists in tracking its progress
  - Basis for inference about degradation and its progression towards a threshold
  - Offers scientific insight into the forces driving degradation
- Example of marker variables
  - In ball bearing fans, the degradation variable may be the surface defect/roughness of the bearing balls. Possible markers:
    - Vibration (Hz)
  - In a laptop computer, the degradation variable(s) may not be known. Possible markers:
    - Temperature of motherboard
    - Fan speed
    - etc

Data Structure

$$\begin{aligned}
S_i &= \text{failure time for unit } \; i \quad (\textit{latent}) \\
T_i &= S_i \wedge \tau \;, \quad (\tau = \text{progressive censoring time}) \\
Z_i(t_j \wedge T_i) &= \text{covariates}\,, \; j = 1, \ldots, k \\
X_i(T_i) &= \text{terminal degradation} \;\; (= a \text{ if } \; S_i \leq \tau) \\
Y_i(t_j \wedge T_i) &= \text{marker, longitudinal obs. until terminal time}
\end{aligned}$$

Main issues for this talk:

$Y_i(T_i)$ generally available, $X_i(S_i) = a$ implicit

Augmented data consist of values $X_i(\tau)$ for $\tau < S_i$

and $Y(t_j)$ for $t_j < T_i$

Bivariate Process Model

$(X(t), Y(t))$ bivariate Wiener process, indep. increments

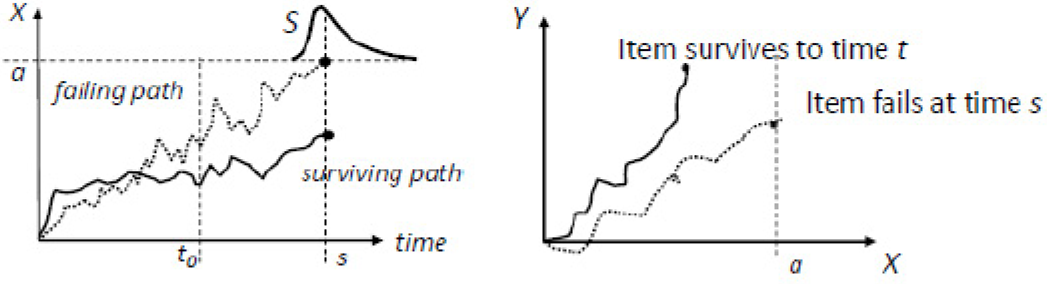
$$X(t) \sim \mathcal{N}(\nu_x,\ \sigma_x^2\, t) \quad , \qquad S \equiv \inf\{t :\ X(t) = a\}$$

$$Y(t) \sim \mathcal{N}(\nu_y,\ \sigma_y^2\, t)$$

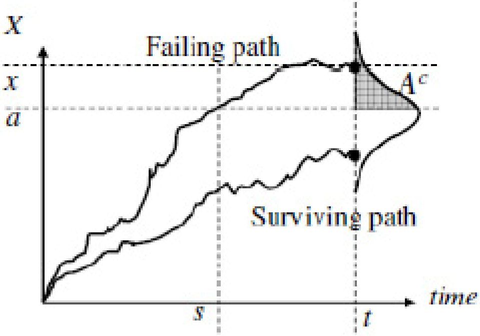
$$Y(t) - \rho\,\frac{\sigma_y}{\sigma_x}\,X(t) \quad \text{indep. of} \quad X(t) \qquad \text{as processes}$$

Model as in Whitmore, Crowder and Lawless (1998); Censored Data and longitudinal data as in Lee and Whitmore (2007).

NB. Normality of increments of $Y(\cdot)$ can be weakened.



Pictures of paths for Degradation Process X in relation to Failure



Reflection Principle & density for $(X(t), I(S > t))$

Say $X(0) = 0 < a,\ s < t,\ \ x < a$, so $a < 2a - x$:

$$f_{X(t)|S}(x|s;\,\nu_x) \;=\; \exp\left((\nu_x/\sigma_x^2)\,(x - \nu_x(t-s)/2)\right) f_{X(t)|S}(x|s;\,0)$$

$$=\; \exp\left((\nu_x/\sigma_x^2)\,(x - \nu_x(t-s)/2)\right) f_{X(t)|S}(2a - x|s;\,0)$$

$$=\; \exp\left((\nu_x/\sigma_x^2)\,(x - \nu_x(t-s)/2)\right) f_{X(t)|S}(2a - x|s;\,0)$$

$$=\; e^{2\nu_x(x-a)/\sigma_x^2}\, f_{X(t)|S}(2a - x|s;\,\nu_x)$$

Integrate product by $f_S(s)\,ds$ over $s < t$ to conclude:

$$f_{X(t),I(S<t)}(x, 1) \;=\; e^{2\nu_x(x-a)/\sigma_x^2}\, f_{X(t)}(2a - x)$$

Parametric Likelihoods

Parameters $(\nu_x,\, \sigma_x,\, \nu_y,\, \sigma_y,\, \rho)$: put $c = \rho\sigma_y/\sigma_x$, $a \equiv 1$

(Terminal-data-only case: $k = 1,\, t_1 = \tau$) $\delta_i \equiv I(S_i \le \tau)$

$$\prod_{i=1}^{n} \, [f_{S,Y(S)}(S_i, Y_i(S_i))^{\delta_i} \, f_{X(\tau),\, Y(\tau),\, I(S_i \le \tau)}(X_i(\tau), Y_i(\tau), 0)^{1-\delta_i}]$$

$f_S(s)$ Inverse Gaussian, $f_{Y(S)|S}(x|s) = f_{Y(s)-ca}(y - ca)$

$f_{X(\tau),Y(\tau),I(S_i \le \tau)}(x, y, 0) =$

$$f_{Y(\tau)-cX(\tau)}(y - cx) \cdot \{f_{X(\tau)}(x) - f_{X(\tau),I(S \le \tau)}(x, 1)\}$$

last density on previous slide via Reflection Principle

Parametric Likelihood, continued

(Longitudinal-data case: $k > 1,\ t_k = \tau$)

$(X(t_j),\ Y(t_j),\ I(S > t_j))$ Markov Sequence, $j = 1, \ldots, k$

$\{Y(t_j) - cX(t_j)\}_{j=1}^{k}$ indep. of $\{(X(t_j),\ I(S > t_j))\}_{j=1}^{k}$

$f_{X(t+b, I(S>t+b) |\, X(t), I(S>t)}(u + x, 1 | u, 1) \; = \; f_{X(b), I(S>b)}(x, 1)$

process re-started at $X(t) = u$: Markov property

Density obtained via Reflection Principle on earlier slide

Information comparisons

Parameters obtained via Likelihood Maximization

approx. large-sample variances obtained as

inverse of empirical Fisher Information $(-\nabla\nabla^{tr}\,\textit{logLik})^{-1}$

Compare: **(TRM)** terminal-marker only $T, Y(T)$

versus: **(TRM+D)** plus terminal-degradation $T, Y(T), X(T)$

vsersus: **(LongM)** plus longitudinal markers $T, Y(t_j \wedge T), X(T)$

Variance & ARE Comparisons

Statistic of interest: expected failure time $\mu_X = a/\nu_x$

Simulated data: $(\nu_x, \sigma_x, \nu_y, \sigma_y) = (.1, .1, 1.0, .4)$, $n = 40$

$R = 500$ replications for **TRM**, **LongM**, k=2, and **TRM+D**

Varied ρ, τ for effect on Variance-ratio, **LongM** vs. **TRM+D**
(Here $k = 2$, $t_1 = \tau/2$, $t_2 = \tau$.)

rho	tau= 4	7	10	13
0	.999	1.000	1.000	1.000
0.3	.987	0.993	0.997	1.001
0.6	.934	0.961	0.983	1.002
0.9	.803	0.856	0.936	0.987

On next slide: **TRM+D** vs. **TRM**

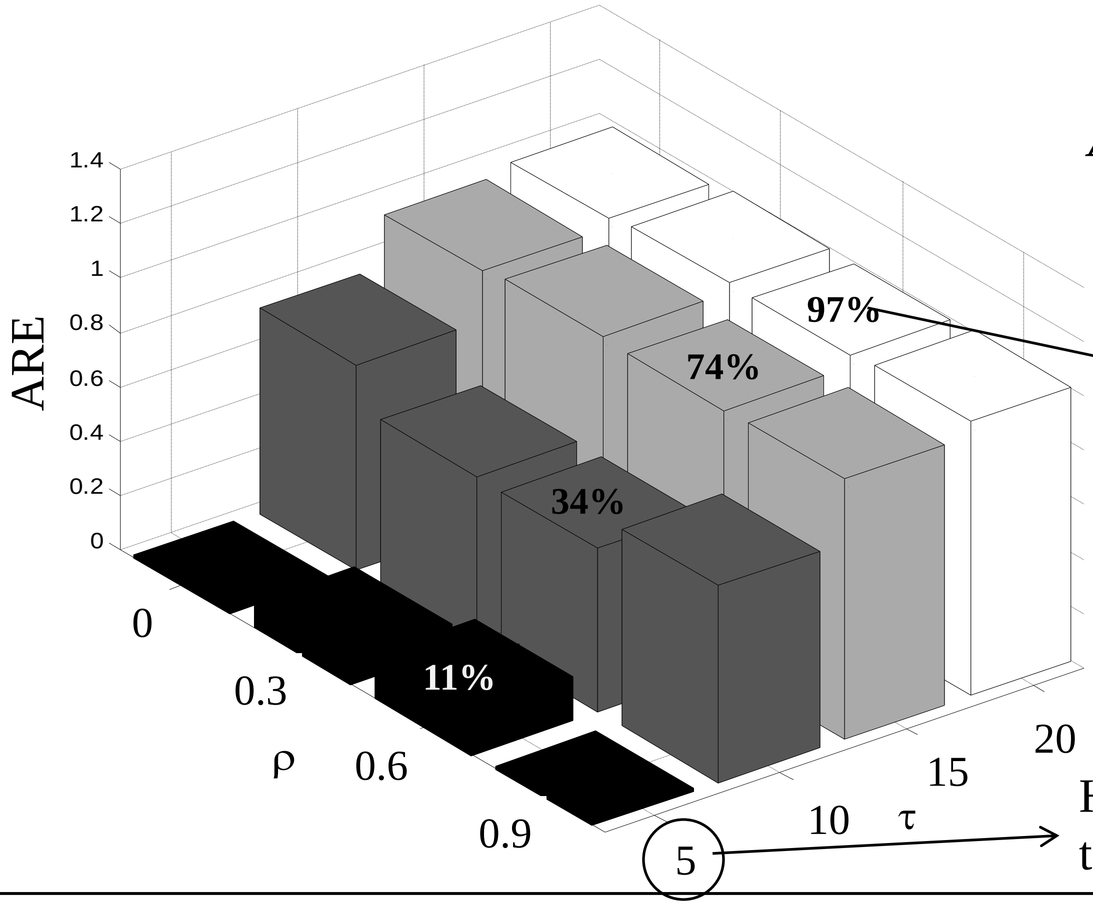
Relative Efficiency

- A more quantitative measure of comparison is the asymptotic relative efficiency *(ARE)* in estimating μ

NEW model

$$ARE = \frac{SE(\hat{\mu}_1)}{SE(\hat{\mu}_2)}$$

Existing Model



Proportion of Failed devices

High ARE when test-time is short

Predictive Inference

Consider two types of predictive inference equations that exploit marker information, the second being of primary interest

Maximum likelihood estimate (MLE) vector of model parameters:

$\hat{\theta} = (\hat{\sigma}_X, \hat{\sigma}_Y, \hat{\nu}_X, \hat{\nu}_Y, \hat{\rho}, \hat{a})$

Prediction of the degradation level: X

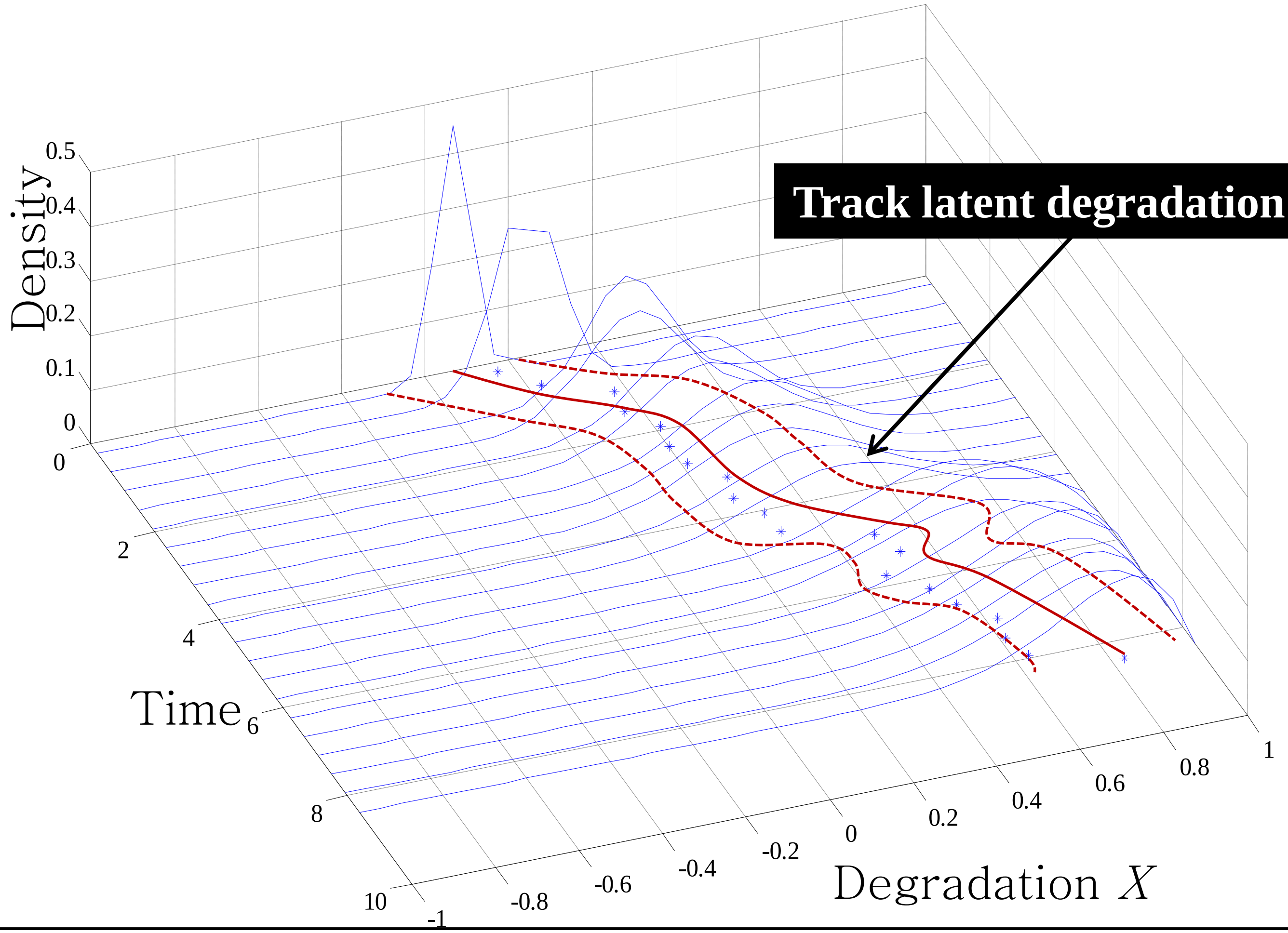
$$P(X(t) = x | Y(t) = y, S > t) = g(\hat{\theta}; t) \tag{1}$$

Prediction of failure time: S

$$P(S = s | Y(t) = y, S > t) = h(\hat{\theta}; t) \tag{2}$$

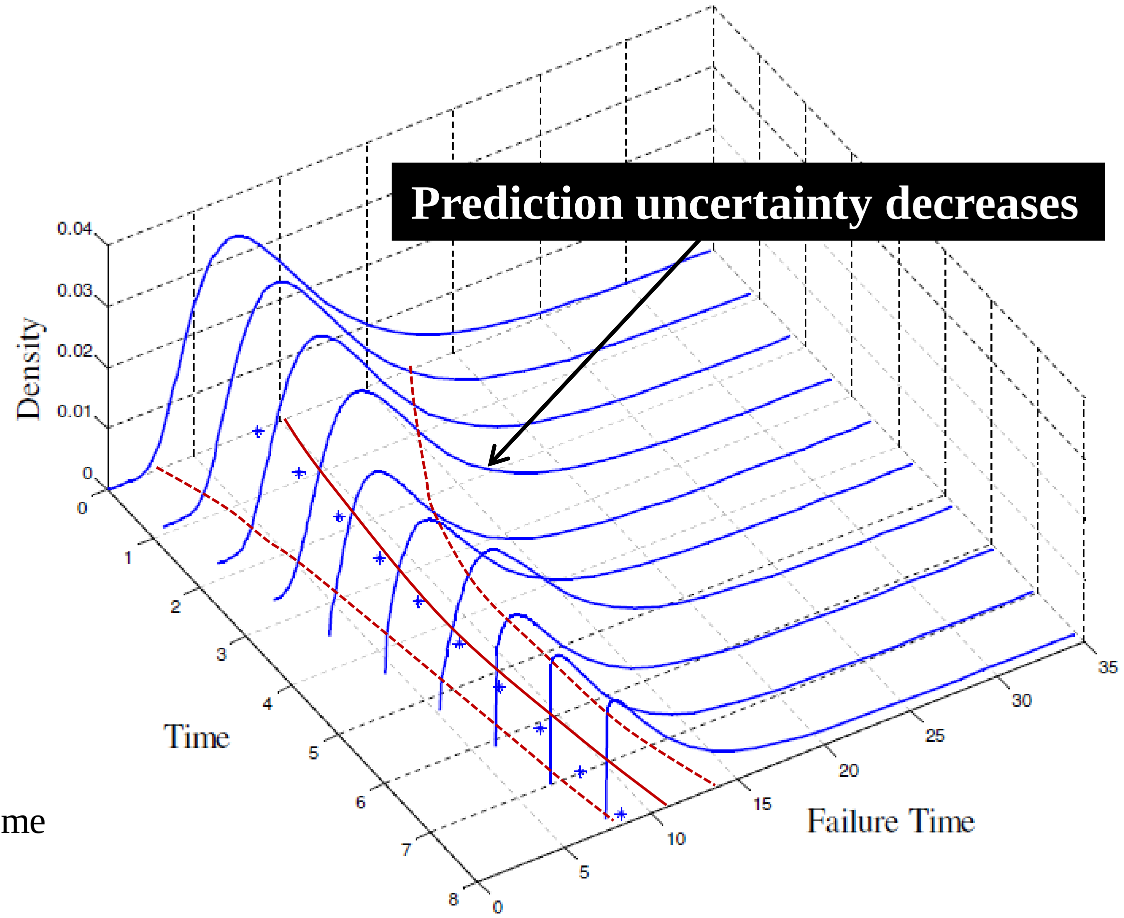
Prediction Results

- For test device surviving at time *t* given a marker observation $y(t)=y$, we predict its degradation distribution


Track latent degradation process

Prediction Results

- For test device surviving at time *t* given a marker observation *y(t)=y*, we predict its future failure-time distribution



Prediction uncertainty decreases

* Actual failure time

Directions of Future Work

- systematic examination of variances of parameter estimators on ρ and τ, for **TRM** vs. **TRM+D** vs. **LongM** data-types.

- analogous models, estimates, and variance comparisons with *random thresholds* a.

- regression models based on external covariates Z_i for ν_x, ν_y.

- other distributional forms of independent-increments processes $Y(\cdot) - c\,X(\cdot)$ indep. of $X(\cdot)$

- possibility of nonlinearly transforming marker measurements to make these models fit better.

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