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Office hrs. M11, W4

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Class-time MW 5:15–6:30pm
Classroom MTH 0307
http://www.math.umd.edu/ evs/s700

Stat 700, Fall 2014

This course introduces mathematical statistics at a theoretical graduate level, using tools of advanced calculus and basic analysis. The objectives are to treat diverse statistically interesting models for data in a conceptually unified way; to define mathematical properties which good procedures of statistical inference should have; and to prove that some common procedures have them.

Prerequisite: Stat 410 or equivalent. You should be comfortable (after brief review) with joint densities, (multivariate, Jacobian) changes of variable, moment generating functions, and conditional expectation.

Text: Casella, G. and Berger, R. *Statistical Inference, 2nd ed.* Duxbury/CENGAGE.

This is a good and comprehensive text, with greater emphasis on simulation, computational algorithms (rigorously justified !) and Bayesian methods than some other books at this level like Rohatgi's. Other books such as the recommended texts of Bickel and Doksum and of Shao contain mathematically more difficult (less readable) material, but also cover interesting topics, some of which we will cover from handouts and occasional reference to those books.

Course Coverage: This fall, we will cover the material in Chapters 1–4 fairly rapidly, as review (in about four lectures). After that, we will cover most of Chapters 5-8. We spend some time on Chap. 5 (going lightly over Sections 5.5.1-5.5.3, all other sections thoroughly), covering Chapters 6-7 in full detail and going as far through Chapter 8 as we have time for. This means that we will discuss properties of statistical procedures that are meaningful for small or moderate samples reserving asymptotic techniques for large-sample data until the spring term, and will revisit the Likelihood Ratio Test (going beyond the treatment in Casella and Berger) for large-sample data. Further coverage of Bayes and empirical Bayes methods will be presented in the spring along with material on confidence intervals. Computational topics will be treated both terms, with EM algorithm and other topics related to likelihood maximization and basic simulation in the fall, and bootstrap and Markov Chain Monte Carlo in the spring. Many of these topics will be covered through handouts, during both terms.

Grading: There will be graded homework sets roughly every1.5–2 weeks (about 8 altogether). There will be two in-class tests, the first on Monday October 6, covering material through Chapter 5, and an in-class Final Exam. The course grade will be based 30% on homeworks, 40% on tests, and 30% on the Exam.

Problem Set 1, due Monday, Sept.15: Do problems # 3.20 and 3.34 in Casella-Berger along with

(1). Prove that the logistic density is symmetric about 0 [see exercise #2.26 for definitions and properties of symmetric densities], and show that its mean is 0 and by integration evaluate its variance (as given on p. 624 for $\mu = 0, \beta = 1$) or find a valid series expansion for it.

(2) Suppose that the distribution function of a random variable X is given by:

$$F(t) = (\frac{1}{4} + \frac{t}{12})\, I_{[0 \le t < 1]} \; + \; \frac{1}{3}\,(1 + (t-1)^2)\, I_{[1 \le t < 2]} \; + \; I_{[t \ge 2]}$$

Find the mean, variance, and moment generating function of X.

(3) Verify in two ways — both analytically and by verifying the conditions of Theorem 2.4.3 or 2.4.8 in Casella & Berger — that $dM(t;\lambda)/dt = E(X\exp(tX))$ where the random variable X is Poisson(λ) distributed and $M(t) = M(t;\lambda) = E(\exp(tX))$ denotes its moment generating function.

(4) Prove that the first term on the right-hand side of the last displayed equation on p. 124 (3rd line from the end of the proof of Lemma 3.6.5) is 0, under the conditions of the Lemma.

(5) Prove [the assertion in Casella-Berger problem #2.7(b)] that Theorem 2.1.8 continues to hold if the sets $A_0, A_1, \ldots, A_k$ partition a set **larger** than the set (2.1.7) of positivity of the density f_X. (This means that the disjoint sets A_i are allowed to contain points x where $f_X(x) = 0$.) Use this fact (or other results in Chapter 2 of Casella and Berger) to find the density of $Y = (X-2)^2$, where X is a random variable which is uniformly distributed on $[0, 3]$.

(6) Suppose that a positive-valued random variable T has the survival function $S(t) = 1 - F(t) = \exp(-bt^a)$ for all $t > 0$, for positive constants a, b. Show that for each fixed value of $a > 0$, this family of probability distributions parameterized by $b > 0$ is a `scale family`, and that the distribution of the random variable $\log(T)$ is a `location-scale` family with respect to the parameters (a,b).