STAT 770 Nov. 4 Lecture 19A
Some Loose Ends from Chapters 6-8

Reading and Topics for this lecture: Chapters 6-8.

(1) ROC and AUC, again (pp. 224-225)

(2) Sample Size for X^2 tests (pp. 239-240)

(3) Interpretation of Logistic Regression Parameters as Causal log Odds Ratios

Second half of Lec. 19 will introduce Loglinear Models, Chap. 9

Formal Definition of ROC & AUC (in GLM setting)

Setting: binary outcomes Y_i, GLM predictions $\widehat{\mu}_i$ (continuous)

For threshold t: Diagnostic *predicts* $Y_i = 1$ when $\widehat{\mu}_i \geq t$

$$\mathbf{ROC} : (1-\mathtt{Spec}(t), \mathtt{Sens}(t)) = \left(\frac{\sum_{i=1}^n (1 - Y_i)\, I_{[\widehat{\mu}_i \geq t]}}{\sum_{i=1}^n (1 - Y_i)}, \frac{\sum_{i=1}^n Y_i\, I_{[\widehat{\mu}_i \geq t]}}{\sum_{i=1}^n Y_i} \right)$$

Coords $\downarrow$ in t: discontinuity at sorted $t \in \{0, 1, \{\widehat{\mu}_j\}_{j=1}^n\} = \{a_k\}_{k=0}^m$

AUC: area under ROC by Trapezoid rule

$$\sum_{k=0}^{m-1} \Big(\mathtt{Spec}(a_{k+1}) - \mathtt{Spec}(a_k)\Big) \cdot \frac{\mathtt{Sens}(a_k) + \mathtt{Sens}(a_{k+1})}{2}$$

`R` *package* `ROCR` *can be used with more general binary classifiers*

Power Increasing With Degrees of Freedom

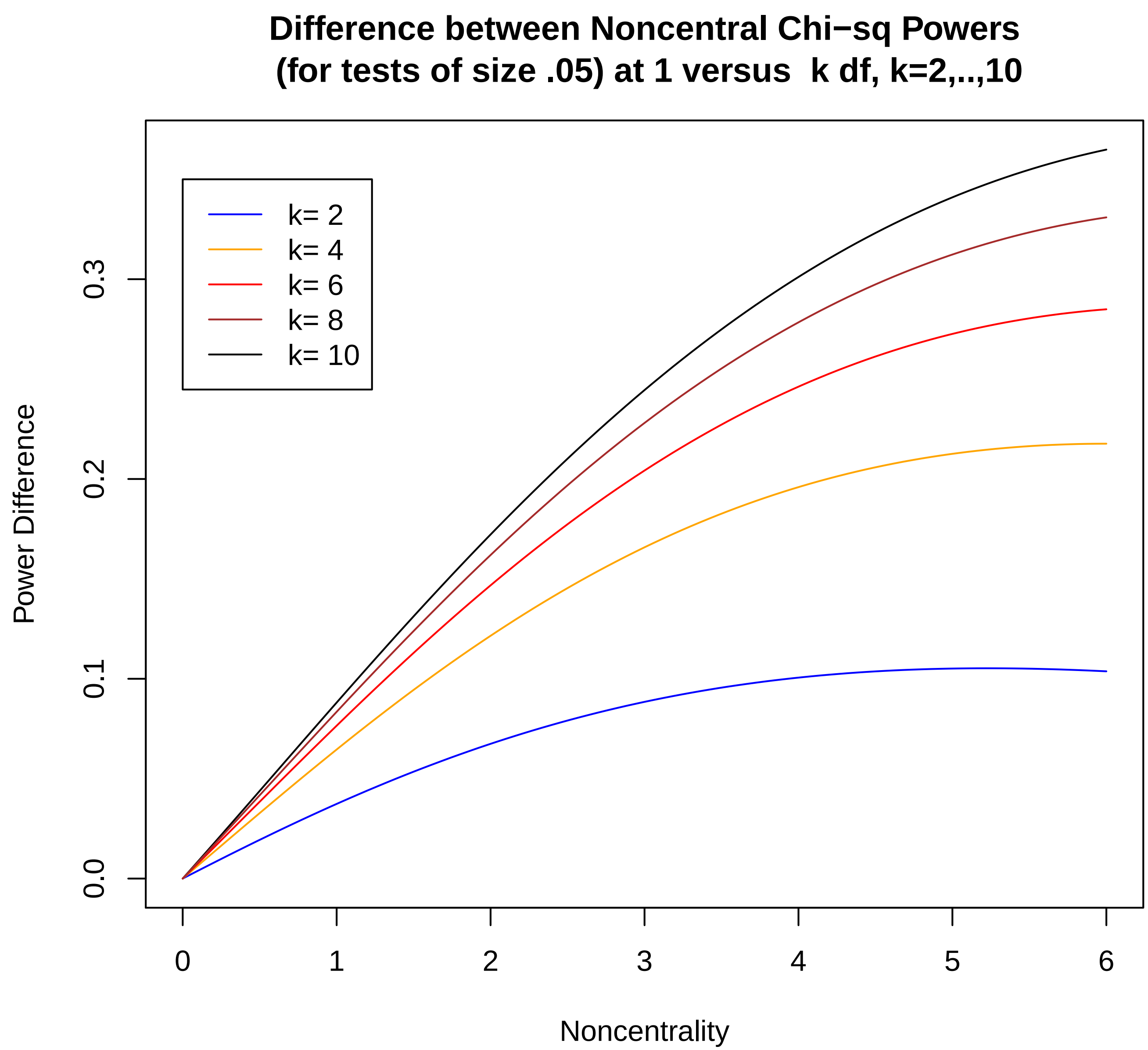
Recall last time that we found that 'focused-alternative' trend-tests were more powerful than general X^2 tests, as expressed by the inequality

$$1 - \texttt{pchisq}(\chi^2_{m,\alpha}, m, \Delta^2) \le 1 - \texttt{pchisq}(\chi^2_{1,\alpha}, 1, \Delta^2)$$

for all $m \geq 2$ and $\Delta^2 \geq 0$ and α.

Here is a picture showing it graphically, from the code

```
curve(pchisq( qchisq(.95, 10), 10, x) -
      pchisq( qchisq(.95,1), 1, x), 0,6)
```



Difference between Noncentral Chi−sq Powers
(for tests of size .05) at 1 versus k df, k=2,..,10
k= 2
k= 4
k= 6
k= 8
k= 10
Power Difference
0.0
0.1
0.2
0.3
0
1
2
3
4
5
6
Noncentrality

Power & Sample-Size for X^2 Contingency-table Tests

General theory from last time shows: if cell-counts N_i have total table-count n and p free $\pi_i = \pi_i(\beta)$ and GLM-estimated $\pi_i(\widehat{\beta}_r)$ under $H_0: \gamma = \mathbf{0}$ with $\dim(\gamma) = q$, then

$X^2 = \sum_i \frac{(N_i - n\pi_i(\widehat{\beta}))^2}{n\pi_i(\widehat{\beta})} \overset{H_0}{\sim} \chi^2_q$ equivalent to Score-Statistic

under $H_{A,n}: \gamma = b/\sqrt{n}$, $X^2 \sim \chi^2_q(b^{tr} D_\gamma b)$, D_γ from last time

For power $1-\delta$ against γ_1: $1 - \delta \le 1 - \texttt{pchisq}(\chi^2_{q,\alpha}, q, n\gamma_1^{tr} D_\gamma \gamma_1)$

Examples: (i) K Multinomial cells, $p = K - 1$, $q = 0$

(ii) $J \times 2$ table with fixed row-totals n_i, $p = J$, $\pi_{j,+} = n_j/n$

Intrpretation of Logistic Regression Coefficients

For binary outcomes Y_i, logistic regression on variables X_i :

Agresti often wants to interpret (estimate of) coeff of β_j of binary component $X_{i,j}$ as logit of logit(μ_i) with $X_{i,j} = 1$ minus logit(μ_i) with $X_{i,j} = 0$, i.e., as a **log odds ratio**.

This is questionable 'causally' if the replacement $0 \mapsto 1$ affects other variables, especially interaction terms $X_{i,j^*} = X_{i,j} \cdot X_{i,j'}$

In that case and more generally, the replacement effect is to change μ_i from $g^{-1}(\beta^{tr} X_i)$ to $g^{-1}(\beta^{tr} X_i^*)$