Math 461, Course review sheet

Reminder: **the final is on May 22 at 8:00 am in math 0303**
These are some of the main topics we've covered as of May 14. This list is not complete and should not be relied on to cover everything that might be on the May 22 final. I would recommend that you go through your notes, the texts and the homework sets and add more topics and more details to this list. I would expect you to be familiar not just with the results but at least some of the main points involved in proving them.

1 first half of the course

- Linear equations and their solution sets.
- The correspondence between linear equations and matrices.
- Using row reduction to solve systems of linear equations. Be able to tell when a system has one solution, no solutions or infinitely many solutions.
- Vector equations and matrix equations. linear combinations and spans.
- Linear independence: understand the definition, be familiar with examples of linearly dependent and independent sets of vectors. Be familiar with the proof in homework 3 and how to use the result to test if a set of vectors in $\mathbb{R}^n$ is linearly dependent.
- Linear transformation and their connection to matrices. You should know how a matrix defines a linear transformation and how a linear transformation defines a matrix. Be familiar with examples of linear transformations and of transformations which are not linear and be able to prove that they are or are not.
- Inverses of matrices. Know the definition of the inverse, how to test if a matrix has an inverse and how to calculate the inverse of a matrix (we learned two ways). Go over the big invertible matrix theorem which lists many equivalent conditions for a matrix to be invertible.
- Determinants and their properties. How do row operations effect the determinant and how to use them to calculate a determinant. Cofactor expansion of the determinant along any row or column.
- Vector spaces. Know the definition of a vector space and a subspace. Be familiar with some examples of vector spaces and subspaces that they have. In the case of $\mathbb{R}^2$ and $\mathbb{R}^3$ you should be familiar with descriptions of subspaces as lines and planes. Be able to describe some subsets of vector spaces which are not subspaces, for example a line in $\mathbb{R}^2$ that does not pass through the origin. Be able to test whether a subset is or is not a subspace.

- Know the definitions of a basis, the dimension of a vector space and the rank of a matrix.
- Vector spaces other than $\mathbb{R}^n$ such as polynomial spaces and function spaces.

2 Second half of the course

- know the definition of eigenvectors and eigenvalues. Know how to calculate both for a square matrix. The eigenvalues values are the roots of the characteristic equation and the eigenvectors v are in the null space of $A - \lambda I$. you should be able to explain facts like that a matrix is invertible is and only if 0 is not an eigenvalue for it. Know the diagonalization theorem on page 314 and theorem 7 on page 318.
- know about dynamical system and their connection with eigenvectors. The geometric picture for eigenvalues with magnitude less than 1 and for eigenvectors with magnitude greater than 1. Know the definition of a stochastic system and how to find the steady state vector and how to interpret it's meaning as describing the long term behavior of the system.
- know the basic definitions and examples of inner products- not just the standard inner product. Be able to use the gram Schmidt process to produce orthogonal (and orthonormal) sets from linearly independent sets of vectors. Be able to calculate projections onto subspaces and to interpret the projection as minimizing distances.
- understand the statement of the least square problem and how projections are applied to the solution. Be able to set up a least square problem from a word problem and solve the system.