CORRECTIONS AND ADDITIONS TO

Nonlinear Problems of Elasticity

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Last modified 1 February 2000

Preface

P xii, L 8. Add "John M. Ball".

Chapter I

P 3, L 19. Replace "by y) that" with "by y), which".

P 3, L –1. Replace "functions" with "real-valued functions".

P 4, L 2. Add the sentence

Note that an inequality of the form $\phi > 0$ is quite different from a statement that ϕ is everywhere positive, i.e., that $\phi(x) > 0$ for all x in the domain of ϕ.

P 4, L –13. Add "(Formal definitions of all these terms are given in Sec. XVII.1.)"

P 4, L –9. Delete "the".

P 4, L –7. Add "The zero vector of $\mathbb{E}^3$ is denoted $\mathbf{0}$."

P 4, L –6. Read"set of three vectors in $\mathbb{E}^3$."

P 5. Two lines after (4.2), after the period insert

(As we shall see when we introduce components, these notations are designed to indicate that the contribution of $\boldsymbol{f}$ to the tensor $\partial \boldsymbol{f}/\partial \boldsymbol{u}$ precedes that of $\boldsymbol{u}$. In particular, $\partial \boldsymbol{f}/\partial \boldsymbol{u}$ does not in general equal $(\partial/\partial \boldsymbol{u})\boldsymbol{f}$, which denotes its transpose in the notation introduced in Chap. XI.) It follows from this definition that if $\boldsymbol{g}$ is Fréchet differentiable near $\boldsymbol{v}$ and if $\boldsymbol{w} \mapsto \boldsymbol{f}(\boldsymbol{w})$ is Fréchet differentiable near $\boldsymbol{g}(\boldsymbol{v})$, then the composite function $\boldsymbol{u} \mapsto \boldsymbol{f}(\boldsymbol{g}(\boldsymbol{u}))$ is Fréchet differentiable near $\boldsymbol{v}$, and its Fréchet derivative is given by the Chain Rule:

$$\frac{\partial \boldsymbol{f}(\boldsymbol{g}(\boldsymbol{u}))}{\partial \boldsymbol{u}} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{w}}(\boldsymbol{g}(\boldsymbol{u})) \cdot \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{u}}(\boldsymbol{u}).$$

Add page 5 to the entry in the Index for the Chain Rule.

P 6, L 1. Read "Theorem of Calculus and the Chain Rule imply that"

P 6, (4.3). In the second line, replace

$$\frac{\partial}{\partial \mathsf{u}}\mathsf{f} \quad \text{with} \quad \frac{\partial \mathsf{f}}{\partial \mathsf{u}}.$$

P 6, L 6. Read "yield".

P 6. After the second paragraph insert the paragraph

A (*parametrized*) *surface* (*patch*) is a continuous function $(s_1, s_2) \mapsto \boldsymbol{r}(s_1, s_2) \in \mathbb{E}^3$ defined on a region of $\mathbb{R}^2$. The surface $\boldsymbol{r}$ is continuously differentiable iff it admits a parametrization, say with (s_1, s_2), such that $\boldsymbol{r}$ is continuously differentiable with respect to these parameters and such that $\frac{\partial \boldsymbol{r}}{\partial s_1} \times \frac{\partial \boldsymbol{r}}{\partial s_2} \neq \mathbf{0}$.

P 8, first line of the fourth paragraph. Put a comma after "(*Lebesgue-*) *measurable*".

P 9, L 1. Insert a comma after " ... ".

P 9, L –5. Add "at most" after "differing".

Chapter II

P 18, L 2. Replace "*differential*" with "*strain-rate*". Replace the sentence in parentheses with "(Some authors refer to such materials as being of *rate type*, while others refer to them as being of *differential type*, reserving *rate type* for an entirely different class.)"

P 19, L 6,7. Replace "we may produce ... , the integral of which" with "we may produce a stretch that varies from point to point; the integral of the stretch".

P 28. In (5.5) and the next line, replace σ with γ.

P 31, L –7. Put a comma after $\hat{N}_s = 0$.

P 33, L –1. Delete the period and add "where z is the solution of the equilibrium problem given in Sec. 6. For well-behaved solutions, the inital data should satisfy the compatibility conditions $\boldsymbol{u}_1(0) = \mathbf{0} = \boldsymbol{v}_1(0)$, $\boldsymbol{u}_1(1) = \mathbf{0} = \boldsymbol{v}_1(1)$."

Chapter III

P 58. After the sentence containing (3.22), insert "Note that this definition ensures that there are at least as many solutions of (3.21) as $|\deg(\mathsf{f}(\cdot, a), \Omega)|$."

P 66, two lines after (5.1). Do not split the parenthesis.

P 81 (7.23). Replace ξ with N.

P 82. The argument encompassed in (7.31)–(7.34) is faulty in the absence of stronger constitutive restrictions that allow the use of Fatou's Lemma. An alternative approach is embodied in Exercise 7.35.

Chapter IV

P 88, one line after (1.6). Delete one "that".

P 93, one line after (1.33b). Add a * to the first W.

P 103. After (3.11) add: "Note that the value of κ is the curvature of $\boldsymbol{r}$ for an unshearable ring."

P 104. In the line following (3.16b) delete "with".

P 122, (5.40). Replace la with λ

Chapter V

P 126, (1.3). Replace $\boldsymbol{r}, \boldsymbol{a}, \theta$ with $\mathring{\boldsymbol{r}}, \mathring{\boldsymbol{a}}, \mathring{\theta}$.

P 129. Near the end of the paragraph, replace "three solution pairs" and "five solution pairs" with "at least three solution pairs" and "at least five solution pairs".

P 130, L –14. Replace "Fig. 1.12" with "Fig. 1.15".

P 140, one line after (3.1). Replace $\mathcal{X} \times \mathbb{R}^n$ with $\mathbb{R}^n \times \mathcal{X}$.

P 146, L 18. Replace "trivia" with "trivial".

P 152, L 3. Replace "be Fréchet-differentiable" with "have Fréchet derivative $\mathsf{A}(\boldsymbol{\lambda})$".

P 152, one line after (4.5). Read "bounded linear operator".

P 155, L –10. Replace (3.22) with (III.3.22).

P 160, 161. Replace the M used for a bound with K. Replace g_k with γ_k.

P 161, Ex. 5.6. Replace the last three sentences with "Suppose that the rod models a three-dimensional body $\{(x, y, s) : 0 \leq s \leq 1, -h(s) \leq x \leq h(s), -b \leq y \leq b\}$ with a rectangular cross section. Here b is a given positive constant and h is a given function with $h(s) > 0$ for $0 < s < 1$. It is standard engineering practice to take $EI(s)$ to be the product of a constant (elastic modulus) E and the areal moment of inertia I of the section s: $I = \int_{-b}^{b} \int_{-h(s)}^{h(s)} x^2 \, dx \, dy$. (See Chap. XIV for a detailed discussion of the generation of rod theories from three-dimensional theories.) If the the rod has thickness $h(s)$ =const$(1-s)^\alpha$ near $s = 1$, find the critical α separating compact from noncompact operators."

P 161, Ex. 5.7. Replace " lower bound" with "upper bound".

P 161, L –4. Replace $M(\cdot, s)$ with $\hat{M}(\cdot, s)$.

P 161, L –6. Replace λ_k with λ_k^0.

P 163, L –5. Delete "the closure of".

P 164, L -9. Read "pair in". Replace "nonzero" with "nontrivial".

P 167, three lines after (6.6). Replace $\lambda_{(0)}$ with $\lambda_{(1)}$.

P 169, L –3. Replace "define the amplitude ε as" with "take the amplitude ε to be".

P 170. After L 5, add the following paragraph:

It is not immediately evident that (6.21) is consistent. We show that it is for the general problem (3.1) when $\mathcal{X}$ is a Hilbert space. Suppose that $(\partial \mathbf{f}/\partial \mathbf{u})^0$ (cf. (6.3)) has a one-dimensional null space spanned by the unit vector $\mathbf{u}_1$. Then (6.21) is just a special version of

$$\varepsilon - \langle \mathbf{u}(\cdot, \varepsilon), \mathbf{u}_{(1)} \rangle = 0. \tag{6.23}$$

If this equation were to be used as a definition of ε, then we would hope that we could solve it uniquely for ε by the Implicit-Function Theorem. But in view of (6.1b), the derivative of the left-hand side of (6.23) with respect to ε at $\varepsilon = 0$ is just $1 - \langle \mathbf{u}_{(1)}, \mathbf{u}_{(1)} \rangle = 0$. To circumvent this difficulty, we introduce the parameter ε explicitly into our bifurcation problem:

$$\mathbf{f}[\lambda, \mathbf{u}] = \mathbf{0}, \quad \varepsilon - \langle \mathbf{u}, \mathbf{u}_{(1)} \rangle = 0. \tag{6.24a}$$

We abbreviate this system as

$$\mathbf{g}[\lambda, \mathbf{u}, \varepsilon] = \mathbf{0}. \tag{6.24b}$$

Now $\mathbf{g}[\lambda, \mathbf{0}, 0] = \mathbf{0}$. We seek to solve (6.24) for λ and $\mathbf{u}$ as functions of ε for $(\lambda, \mathbf{u}, \varepsilon)$ near $(\lambda_{(0)}, \mathbf{0}, 0)$. If $\mathbf{f}$ is nice enough, then Theorem XVII.2.34 and the Implicit-Function Theorem XVIII.1.27 say that we can find such a solution if the operator $(\partial \mathbf{g}[\lambda_{(0)}, \mathbf{0}, 0]/\partial\lambda), \partial \mathbf{g}[\lambda_{(0)}, \mathbf{0}, 0]/\partial \mathbf{u})$ is nonsingular, and that this operator is nonsingular if the equation

$$\frac{\partial \mathbf{g}[\lambda_{(0)}, \mathbf{0}, 0]}{\partial \lambda}\mu + \frac{\partial \mathbf{g}[\lambda_{(0)}, \mathbf{0}, 0]}{\partial \mathbf{u}} \cdot \mathbf{v} = \mathbf{0} \tag{6.25a}$$

has only the trivial solution $(\mu, \mathbf{v}) = (0, \mathbf{0})$. But (6.25a) is equivalent to

$$\frac{\partial \mathbf{f}[\lambda_{(0)}, \mathbf{0}, 0]}{\partial \mathbf{u}} \cdot \mathbf{v} = \mathbf{0}, \quad \langle \mathbf{v}, \mathbf{u}_{(1)} \rangle = 0, \tag{6.25b}$$

which clearly has only trivial solutions. Thus $(\partial \mathbf{g}[\lambda_{(0)}, \mathbf{0}, 0]/\partial\lambda), \partial \mathbf{g}[\lambda_{(0)}, \mathbf{0}, 0]/\partial \mathbf{u})$ is nonsingular, and (6.24) admits the requisite solution, having as many derivatives with respect to ε as $\mathbf{f}$ has with respect to its arguments. If we denote the solution of (6.24) by $(\lambda(\varepsilon), \mathbf{u}(\cdot, \varepsilon))$, then we see that (6.23) is an identity.

P 170. Renumber Exercises 6.23 and 6.24 as 6.26 and 6.27. After the newly numbered Exercise 6.27 insert the following material:

6.28. Exercise. Find necessary and sufficient conditions on the sufficiently smooth function f with $f(0) = 0$ such that the scalar problem $x = \lambda x - f(x)$ has (i) supercritical, (ii) subcritical, and (iii) transcritical bifurcations.

6.29. Exercise. To see what happens when $\mathbf{f}$ is not sufficiently smooth for the perturbation method to work, plot the bifurcation diagram for the scalar problem

$$\lambda x = f(x) \equiv \begin{cases} 1 - \sqrt{1+x}, & x > 0, \\ -1 + \sqrt{1-x}, & x < 0. \end{cases} \tag{6.30}$$

Note that f is continuously differentiable.

P 170. In the last sentence of Section 6, replace "see" with "cf.".

Chapter VI

P 177, L –6. Delete left parenthesis.

P 179. In the second line of Ex. 2.14, replace $(0, 1]$ with $(0, 1)$.

P 179, L 26, L 28, and Lemma 2.16. Replace (w, z) with (w, x).

P 180. Replace the first sentence of the last paragraph with
"Let $u(s) = \sqrt{s}w(s) = sy(s)$. We examine solutions on the level

$$\|y\| + \|x\| = R.\text{"} \tag{2.23}$$

Replace the last six lines with: " ... given. If (y,x) also satisfies (2.23), then $h(\sqrt{s}y(s),s) \geq \ldots$. Consider the modification of (2.4) obtained by replacing $h(\sqrt{s}y(s),s)$ and ... $\min_s(\rho A)(s)$. We reformulate the differential equations corresponding to (2.4) in terms of (y,x). We apply the Prüfer ... ."

P 184, equation following (2.49). Insert a space before $\leq$.

P 200, (6.1). Delete commas after α. Replace Λ with λ.

Chapter VII

P 229, the line below (1.12). Read "accounts".

P 236, Theorem 3.10. In the third line, replace (3.8) with (3.9). In the third line of the next paragraph, replace "(3.8), which is a central ... (5.11)" with "(3.9), which in the guise of (5.3) is a central hypothesis of Theorem 5.9".

P 254. In the two lines above (7.5) replace "Then (5.14), (5.15) imply" with "Then (5.14) implies". In the second line of (7.5) replace $+\boldsymbol{k} \cdot (\boldsymbol{c} \times \hat{\hat{\boldsymbol{r}}}')$ with $-\boldsymbol{k} \cdot (\boldsymbol{\lambda} \times \hat{\hat{\boldsymbol{r}}}')$ and replace $-\boldsymbol{c} \times \bar{\boldsymbol{r}}'$ with $+\boldsymbol{\lambda} \cdot \bar{\boldsymbol{r}}'$.

P 256. Delete second period in L –20, and insert right parenthesis after "UK.".

Chapter VIII

P 263, (2.18). Replace $\mathbf{c}$ with $\boldsymbol{c}$.

P 264, L 7. Replace "*differential*" with "*strain-rate*". At the end of the first paragraph, replace the period with a colon and add

$$\mathsf{m}(s,t) = \hat{\mathsf{m}}(\mathsf{u}^t(s,\cdot), \mathsf{v}^t(s,\cdot), s), \qquad \text{etc.} \tag{2.23}$$

where, as in the notation of (II.2.15), $f^t(\tau) \equiv f(t-\tau)$ for $\tau \geq 0$.

P 265, (3.1). Replace x with $\mathbf{x}$.

P 267, Fig. 3.6. In the figure replace $\hat{\boldsymbol{p}}$ with $\tilde{\boldsymbol{p}}$ two times. Replace the caption with "Configurations $\tilde{\boldsymbol{p}}(\cdot,\cdot,a,t)$ and $\tilde{\boldsymbol{p}}(\cdot,\cdot,b,t)$ of the sections $\mathcal{B}(a)$ and $\mathcal{B}(b)$ at time t."

P 267, (3.8). Replace $\mathbf{f}(s,t)\,d\xi$ with $\boldsymbol{f}(s,t)\,ds$.

P 270, Ex. 4.8(v). Replace "a and b" with "a^α".

P 280. Bring the discussion on page 420 here. In the second line of Section 7, replace "(2.21) and (2.22)" with "(2.21)–(2.23). We limit our attention to the most general form (2.23)." In the fourth line of this section replace "*elastic material*" with "*material with memory*". Replace (7.1) with

$$\begin{aligned} \boldsymbol{n}(s,t) &= \tilde{\boldsymbol{n}}(\boldsymbol{r}^t(s,\cdot), \boldsymbol{r}^t_s(s,\cdot), \boldsymbol{d}^t_\alpha(s,\cdot), \partial_s \boldsymbol{d}^t_a(s,\cdot), s), \\ \boldsymbol{m}(s,t) &= \tilde{\boldsymbol{m}}(\boldsymbol{r}^t(s,\cdot), \boldsymbol{r}^t_s(s,\cdot), \boldsymbol{d}^t_\alpha(s,\cdot), \partial_s \boldsymbol{d}^t_a(s,\cdot), s). \end{aligned} \tag{7.1}$$

In L –3 and L –2, replace "configuration" with "motion".

P 281. In L 1 and L 2, replace "configuration" with "motion". In L 10, replace $\{\boldsymbol{r}^\sharp, \boldsymbol{r}_s^\sharp, \boldsymbol{d}_\alpha^\sharp, \partial_s \boldsymbol{d}_a^\sharp\}$ with $\{(\boldsymbol{r}^\sharp)^t, (\boldsymbol{r}_s^\sharp)^t, (\boldsymbol{d}_\alpha^\sharp)^t, (\partial_s \boldsymbol{d}_a^\sharp)^t\}$, and in L 11 replace $\{\boldsymbol{r}, \boldsymbol{r}_s, \boldsymbol{d}_\alpha, \partial_s \boldsymbol{d}_a\}$ with $\{\boldsymbol{r}^t, \boldsymbol{r}_s^t, \boldsymbol{d}_\alpha^t, \partial_s \boldsymbol{d}_a^t\}$. Replace (7.3), (7.4), (7.5) with

$$\boldsymbol{n}^\sharp = \tilde{\boldsymbol{n}}((\boldsymbol{r}^\sharp)^t, (\boldsymbol{r}_s^\sharp)^t, (\boldsymbol{d}_\alpha^\sharp)^t, (\partial_s \boldsymbol{d}_a^\sharp)^t, s), \quad \text{etc.} \tag{7.3}$$

$$\boldsymbol{Q} \cdot \boldsymbol{n} = \tilde{\boldsymbol{n}}(\boldsymbol{c}^t + \boldsymbol{Q}^t \cdot \boldsymbol{r}^t, \boldsymbol{Q}^t \cdot \boldsymbol{r}_s^t, \boldsymbol{Q}^t \cdot \boldsymbol{d}_\alpha^t, \boldsymbol{Q}^t \cdot \partial_s \boldsymbol{d}_\alpha^t, s), \quad \text{etc.} \tag{7.4}$$

$$\begin{aligned} \boldsymbol{Q} \cdot \tilde{\boldsymbol{n}}(\boldsymbol{r}^t, \boldsymbol{r}_s^t, \boldsymbol{d}_\alpha^t, \partial_s \boldsymbol{d}_a^t, s) \\ = \tilde{\boldsymbol{n}}(\boldsymbol{c}^t + \boldsymbol{Q}^t \cdot \boldsymbol{r}^t, \boldsymbol{Q}^t \cdot \boldsymbol{r}_s^t, \boldsymbol{Q}^t \cdot \boldsymbol{d}_\alpha^t, \boldsymbol{Q}^t \cdot \partial_s \boldsymbol{d}_\alpha^t, s), \quad \text{etc.} \end{aligned} \tag{7.5}$$

Two lines above (7.6) replace $\boldsymbol{r}$ with $\boldsymbol{r}^t$. Replace (7.6), (7.7), and the intervening line with

$$\tilde{\boldsymbol{n}}(\boldsymbol{r}_s^t, \boldsymbol{d}_\alpha^t, \partial_s \boldsymbol{d}_a^t, s) = \hat{n}_k(\mathsf{u}^t, \mathsf{v}^t, \boldsymbol{d}_\alpha^t, s)\boldsymbol{d}_k, \quad \text{etc.} \tag{7.6}$$

The substitution of (7.6) into (7.5) and the use of (5.11) and Ex. 5.12 yields

$$\hat{n}_k(\mathsf{u}^t, \mathsf{v}^t, \boldsymbol{d}_\alpha^t, s) = \hat{n}_k(\mathsf{u}^t, \mathsf{v}^t, \boldsymbol{Q}^t \cdot \boldsymbol{d}_\alpha^t, s), \quad \text{etc.} \tag{7.7}$$

Replace the two sentences following (7.7) with "Since the $\boldsymbol{d}_\alpha$ are orthonormal, the $\boldsymbol{Q} \cdot \boldsymbol{d}_\alpha$ are completely arbitrary within this class, so that the $\hat{n}_k$ must be independent of their penultimate arguments. Thus we have proved

7.8. Theorem. *The most general constitutive functions of the form* (7.1) *invariant under rigid motions are* (2.23). *In particular,* (2.21) *and* (2.22) *are respectively the most general constitutive functions invariant under rigid motions for simple elastic and viscoelastic special Cosserat rods.*

Exactly the same proof works for nonsimple materials in which we add to the arguments of (7.1) a finite number of s-derivatives of the arguments present in (7.1). Suppose we add just $\boldsymbol{r}_{ss}^t$ and $\partial_{ss} \boldsymbol{d}_\alpha^t$. Then in place of of (2.23) we would get $\boldsymbol{m}(s,t) = \hat{\boldsymbol{m}}(\mathsf{u}^t(s,\cdot), \mathsf{v}^t(s,\cdot), \mathsf{u}_s^t(s,\cdot), \mathsf{v}_t^t(s,\cdot), s)$, etc. The resulting theory would be inadequate, however. The Principle of Virtual Power for such a theory would require force and moment resultants dual to the variables $\boldsymbol{r}_{ss}$ and $\partial_{ss} \boldsymbol{d}_\alpha$. The constitutive equations for these resultants would be treated the same way. Rod theories with constitutive equations that do not describe simple materials arise naturally in the treatment of incompressible rods in Chap. XIV.6B.

Put the statement and proof of Cauchy's Representation Theorem 7.8 in Section 9. Relegate the proofs of Theorems 7.13 and 7.30 based on this theorem, which are but special cases of the new Theorem 7.8, to exercises in Section 9.

P 287. In the paragraph containing (8.6) replace the bold italic symbols with bold sanserif symbols.

P 292. At the end of Section 8 add:

8.42. Problem. State and prove an analog of Theorem II.7.8 for naturally straight elastic rods.

P 299. Replace the second equation of (10.4) with

$$\hat{m}_3(\mathsf{u}, \mathsf{v}, s) = D(s)u_3$$

and replace the next line with "where E is the (effective) elastic modulus and D is the torsional rigidity found from the St. Venant theory of torsion in linear elasticity. For rods with a circular cross section $D(s) = GJ_{\alpha\alpha}(s)$ where G is the (effective) shear modulus. In linear ... ". Put a period at the end of (10.5) and delete the phrase "where ... modulus." in the next line.

P 300, Exercise 10.6. Put a period after "10.6". Remove the period at the end of (10.7). After (10.7) add "where the J's are constants.".

P 300, Figure 11.2. Replace $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{e}_1, \boldsymbol{e}_2$ with $\boldsymbol{j}_1, \boldsymbol{j}_2, \boldsymbol{j}_3, \boldsymbol{k}_1, \boldsymbol{k}_2$. The part of the light-gray plane lying above the white plane and below the dark-gray plane should appear in dark gray.

P 305, (12.14). Replace the last summand in the first equation with $-\sin\psi(t)\sin\phi(t)\boldsymbol{e}_1(t)$.

P 316, L2, L3, L4. Replace "normal" with "transversal".

P 319. five lines above (16.12) read "usually". Four lines above (16.12) replace $EJ_{\alpha\alpha}$ with $GJ_{\alpha\alpha}$. After Exercise 16.13 insert the following paragraph:

Purely torsional motion. Let $\boldsymbol{k}$ be horizontal and let $\boldsymbol{i}$ point downward. Consider a rod with a naturally straight axis so constrained that its axis lies along the $\boldsymbol{k}$-axis. We seek motions such that $\boldsymbol{r}_s = \boldsymbol{d}_3 = \boldsymbol{k}$. Then $\boldsymbol{d}_1, \boldsymbol{d}_2$ have the form

$$\boldsymbol{d}_1 = \cos\psi\boldsymbol{i} + \sin\psi\boldsymbol{j}, \quad \boldsymbol{d}_2 = -\sin\psi\boldsymbol{i} + \cos\psi\boldsymbol{j}.$$

Exercise. Show that such motions are governed by an equation of the form

$$\boldsymbol{k}\cdot\boldsymbol{m}_s - c\rho A g\sin\psi = \rho J_{\gamma\gamma}\psi_{tt} \tag{A}$$

where g is the acceleration of gravity and $c(s)$ is the distance of the mass center of the section s from $\boldsymbol{r}(s)$ (when (4.4)–(4.7) hold under the assumption (4.2g)). For an elastic rod, show that the constitutive function for $\boldsymbol{k}\cdot\boldsymbol{m}$ reduces to a function of ψ_s and s. (If (8.2) holds, then (A) is typically a quasilinear hyperbolic equation. If the material of the rod is uniform and if $\boldsymbol{k}\cdot\boldsymbol{m}$ depends linearly on ψ_s, then (A) reduces to the *sine-Gordon equation*.)

P 319. Begin the penultimate paragraph with "**Rotatory inertia**.".

P 321. At the end of the paragraph preceding that containing (16.27), add "System (16.23) constitutes Timoshenko's (1921) beam theory, which is usually presented as (16.26) with rotatory inertia neglected." Add reference to index.

Chapter IX

P 325, Exercise 1.4. Read " transversely isotropic".

P 335, L10. After $m^\times = n^\times$ add "(cf. (1.5))". L 11. Replace (1.11c) with (1.11d).

Chapter X

P 345. L 9. Read "$\boldsymbol{n}_2(s, \phi_0)$ and $\boldsymbol{m}_2(s, \phi_0)$".

After (1.8) add:

"Clearly $\boldsymbol{m}_2$ should not have a component in the $\boldsymbol{e}_2$-direction: Its effect on bounding curves of longitude of a curvilinear rectangle would be to bend these curve in the opposite senses and thereby destroy the axisymmetry. The reason $\boldsymbol{m}_2$ should not have a component in the $\boldsymbol{b}$ direction is given in Section XIV.13: $\boldsymbol{m}_2$ can be regarded as a cross product of $\boldsymbol{b}$ with another vector."

In the third line of the next paragraph replace $\boldsymbol{e}_2(s, \phi)$ with $\boldsymbol{e}_2(\phi)$. Add: "The body couple has no other components because the effect on a curvilinear rectangle of a component about $\boldsymbol{a}$, thought of as nearly parallel to $\boldsymbol{r}_s$, would tend to to push one boundary curve of longitude inward and the other outward, while a component about $\boldsymbol{b}$, thought of as nearly perpendicular to $\boldsymbol{r}_s$, would tend to push one bounding curve of longitude upward and the other downward, in each case destroying the axisymmetry. The unsuitability of the absent components can be demonstrated analytically by retaining them and then showing that the equilibrium equations obtained force these components to vanish."

P 346. In the last line of (1.10) read $\boldsymbol{e}_2(\psi)$.

P 347. Add a paragraph to the end of Section 1:

The basic ideas of this section, refined to handle geometric complications and the notion of stress, are used to treat non-axisymmetric problems in Section XIV.13.

Chapter XI

P 371, L 2. Replace IV.1 with VIII.1.

P 371, two lines above (1.1a). Insert "uniquely" after "represented".

P 372. In the line following (1.6b) replace "(1.6b)" with "(1.6a)".

P 373, one line above Proposition 1.12. Insert "unique" before "solution".

P 374. Replace the sentence containing (1.17) with "A tensor can be defined by stating what it does to three independent vectors. We easily find that the tensor taking $\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3$ to $\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3$ respectively is the sum $\boldsymbol{b}_k\boldsymbol{a}^k$ of dyads. It follows from this remark or from (1.3) that if $\{\boldsymbol{a}_k\}$ is a basis, then

$$\boldsymbol{I} = \boldsymbol{a}_k\boldsymbol{a}^k = \boldsymbol{a}^k\boldsymbol{a}_k.\text{"} \tag{1.17}$$

P 374. Replace the statement of Exercise 1.18 with "Given a dyadic basis $\{\boldsymbol{a}_k\boldsymbol{a}_l\}$ for Lin, construct from it bases for Sym and Skw. (Bases for Sym and Skw constructed from a general dyadic basis $\{\boldsymbol{a}_k\boldsymbol{b}_l\}$ for Lin are of no utility in our work.)" In the line preceding (1.19) before "defining" insert "requiring it to be linear and by".

P 374, one line below (1.19). Add "(An invariant definition of trace is given below in (1.40) and (1.41). An alternative invariant definition of trace is that $\operatorname{tr}\boldsymbol{U}$ is $\varphi(\boldsymbol{U})$

where φ is a linear mapping from Lin to $\mathbb{R}$ that is isotropic, i.e., $\varphi(\boldsymbol{Q}\cdot\boldsymbol{U}) = \varphi(\boldsymbol{U})$ for all orthogonal $\boldsymbol{Q}$. This idea is developed implicitly in Theorem XII.13.9.)" In the line preceding (1.20a) before "defining" insert "requiring it to be linear and symmetric and by".

P 374. At the bottom of the page, add:

Exercise. There are many ways to choose norms for tensors. The choice (1.20c), sometimes termed the *Frobenius norm*, is compatible with the inner product ':' on Lin. A common alternative choice, which is induced by the norm $|\cdot|$ on vectors and whose generalizations are widely used in linear-operator theory, is

$$||\boldsymbol{A}|| \equiv \sup_{\boldsymbol{x}\neq 0} \frac{|\boldsymbol{A}\cdot\boldsymbol{x}|}{|\boldsymbol{x}|}.$$

Show that

$$||\boldsymbol{A}|| = \sup_{|\boldsymbol{y}|=1} |\boldsymbol{A}\cdot\boldsymbol{y}|.$$

Use the method of Lagrange multipliers to find equations for the $\boldsymbol{y}$ that maximizes $|\boldsymbol{A}\cdot\boldsymbol{y}|^2$ subject to $|\boldsymbol{y}|^2 = 1$. Characterize $||\boldsymbol{A}||$ as an eigenvalue of an appropriate tensor.

P 375, L 18, 19. Replace "a nonlinear mapping" with "nonlinear mappings".

P 375, (1.27a). Replace $(\boldsymbol{uv})$ with $\boldsymbol{uv}$.

P 377, L 1,2. Replace first sentence with "To compensate for its ugliness, the following notational scheme is convenient for handwriting and has certain conceptual advantages:"

P 377. At the end of the line following (1.36) add "When this notation is used, there is no need to assign to each kind of tensor its own font."

P 377, two and three lines after (1.37). Replace "parallelipiped" with "parallelepiped".

P 378. After (2.2) insert "Since $\boldsymbol{B}$ is symmetric, only the symmetric part of $\partial\vartheta(\boldsymbol{A})/\partial\boldsymbol{U}$ enters $[\partial\vartheta(\boldsymbol{A})/\partial\boldsymbol{U}] : \boldsymbol{B}$. It is appropriate to regard $\partial\vartheta(\boldsymbol{A})/\partial\boldsymbol{U}$ as symmetric. Whether or not we do so, its components with respect to a basis for Sym of the form $\{\frac{1}{2}(\boldsymbol{a}_k\boldsymbol{a}_l + \boldsymbol{a}_l\boldsymbol{a}_k)\}$ form a symmetric matrix. (We get these components by replacing $\boldsymbol{B}$ in (2.2) with $\frac{1}{2}(\boldsymbol{a}_k\boldsymbol{a}_l + \boldsymbol{a}_l\boldsymbol{a}_k)$.)"

P 378, (2.4). Replace the second term with $\frac{\partial}{\partial\boldsymbol{u}}(\boldsymbol{u}\cdot\boldsymbol{u})$.

P 379. Replace the text following (2.6a) with "Now the term in braces is clearly orthogonal to $\boldsymbol{v}$ and $\boldsymbol{w}$. Since (2.6a) says that it is orthogonal to $\boldsymbol{u}$, it is orthogonal to a basis and therefore vanishes:" Replace (2.7b) and the following sentence with

$$\partial \det \boldsymbol{A}/\partial\boldsymbol{A} = (\det\boldsymbol{A})\boldsymbol{A}^{-*} = \operatorname{cof}\boldsymbol{A}. \tag{2.7b,c}$$

Equation (2.7b) is our desired representation. By interchanging $\boldsymbol{A}$ and $\boldsymbol{A}^*$ in (2.7c), we get a representation for $\boldsymbol{A}^{-1}$, which is Cramer's Rule.

P 380, (2.10). Replace the second and third lines of this equation with

$$[\partial(\boldsymbol{A}\cdot\boldsymbol{U})/\partial\boldsymbol{U}] : \boldsymbol{B} = \boldsymbol{A}\cdot\boldsymbol{B}, \quad [\partial(\boldsymbol{U}\cdot\boldsymbol{A})/\partial\boldsymbol{U}] : \boldsymbol{B} = \boldsymbol{B}\cdot\boldsymbol{A},$$
$$[\partial(\boldsymbol{U}^*\cdot\boldsymbol{U})/\partial\boldsymbol{U}] : \boldsymbol{B} = \boldsymbol{B}^*\cdot\boldsymbol{U} + \boldsymbol{U}^*\cdot\boldsymbol{B}.$$

P 380, one line above (2.11b). After "positive-definite" add "and symmetric,".

P 380. Replace (2.11b) with

$$(2.11) \qquad [\boldsymbol{U} \cdot \boldsymbol{U_C} : \boldsymbol{B}]^* + \boldsymbol{U} \cdot \boldsymbol{U_C} : \boldsymbol{B} = \boldsymbol{B} \quad \forall\, \boldsymbol{B} \in \text{Sym}.$$

Replace "Sidoroff (1978)" with " T. C. T. Ting (1996)"

P 380, (2.13). Replace the second term with

$$\frac{\partial(\boldsymbol{f} \cdot \boldsymbol{b}^k)}{\partial u^l}.$$

P 382, two lines above (3.1). Replace $\hat{\boldsymbol{z}}(\mathbf{x})$ with $\tilde{\boldsymbol{z}}(\mathbf{x})$.

P 382. In the line preceding (3.1) insert before "Then" the sentence "Let $\tilde{\mathbf{x}}$ be the inverse of $\tilde{\boldsymbol{z}}$."

P 383, (3.11a). Replace the second term with $(w^p_{,m} + w^k \Gamma^p_{km})\boldsymbol{g}_p$.

P 383. After (3.12) add "We can derive (3.12) by an alternative procedure: For any function $\mathbb{E}^3 \ni \boldsymbol{z} \mapsto \boldsymbol{f}(\boldsymbol{z}) \in \mathbb{E}^3$, set $\tilde{\boldsymbol{f}}(\mathbf{x}) = \boldsymbol{f}(\tilde{\boldsymbol{z}}(\mathbf{x}))$. Then the chain rule implies that

$$\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{z}} = \frac{\partial \tilde{\boldsymbol{f}}}{\partial x^k} \frac{\partial \tilde{x}^k}{\partial \boldsymbol{z}} \quad \text{whence} \quad \nabla \boldsymbol{f} = \frac{\partial \tilde{x}^k}{\partial \boldsymbol{z}} \frac{\partial \tilde{\boldsymbol{f}}}{\partial x^k} \equiv \boldsymbol{g}^k \frac{\partial \tilde{\boldsymbol{f}}}{\partial x^k},$$

from which (3.12) follows."

Chapter XII

P 385, L –13. Replace "a region" with "regions".

P 385, L –5. Replace "that a body have volume ... body is measurable" with "that the reference configuration of a body have volume and that a body have mass and can sustain forces can be expressed by saying that a body is measurable".

P 386. After Exercise 1.2 insert the following one-sentence paragraph:

It is not difficult to show that if $\boldsymbol{p}(\cdot, t)$ is a homeomorphism (i.e., a one-to-one continuous mapping with a continuous inverse) from $\operatorname{cl} \mathcal{B}$ to $\boldsymbol{p}(\operatorname{cl} \mathcal{B}, t)$, then $\boldsymbol{p}(\partial \mathcal{B}, t) = \partial \boldsymbol{p}(\mathcal{B}, t)$.

P 388, L –3. Replace $\boldsymbol{a}_1$ with $\boldsymbol{a}_1(\boldsymbol{z})$.

P 389, Fig. 2.9. Replace θ_* with $\mathring{\theta}$.

P 390, L 7. After $\boldsymbol{n}_3$ insert "of (2.14b)".

P 390, first line of Ex. 2.18. Before "There", insert "Let $\boldsymbol{C}$ be given."

P 390, L –11, -10. Replace the sentence "Any ... " with "In place of $\boldsymbol{C}$, any invertible tensor-valued function of $\boldsymbol{C}$ can be used as a measure of strain."

P 394. Delete the two lines following (4.4). Delete the end-of-proof sign $\square$ in L -8 and continue the proof with the following paragraph:

To prove (4.2b) we apply (4.2a) to $\boldsymbol{F}^*$ obtaining the unique decomposition $\boldsymbol{F}^* = \boldsymbol{P}^* \cdot \boldsymbol{V}$ with

$$(4.5) \qquad \boldsymbol{V} = (\boldsymbol{F}^{**} \cdot \boldsymbol{F}^*)^{1/2} = (\boldsymbol{F} \cdot \boldsymbol{F}^*)^{1/2}, \quad \boldsymbol{P}^* = \boldsymbol{F}^* \cdot \boldsymbol{V}^{-1}.$$

Thus $\boldsymbol{F} = \boldsymbol{V} \cdot \boldsymbol{P}$. We must prove that $\boldsymbol{P} = \boldsymbol{R}$. Equating our two representations for $\boldsymbol{F}$, we obtain $\boldsymbol{R} \cdot \boldsymbol{U} = \boldsymbol{V} \cdot \boldsymbol{P}$, so that $\boldsymbol{R} \cdot \boldsymbol{U} \cdot \boldsymbol{R}^* = \boldsymbol{V} \cdot \boldsymbol{P} \cdot \boldsymbol{R}^*$. Since $\boldsymbol{R} \cdot \boldsymbol{U} \cdot \boldsymbol{R}^*$ is symmetric and positive-definite, it has a unique polar decomposition (of the form (4.2b)) with the rotation the identity. Thus $\boldsymbol{R} \cdot \boldsymbol{U} \cdot \boldsymbol{R}^* = \boldsymbol{V}$, $\boldsymbol{P} \cdot \boldsymbol{R}^* = \boldsymbol{I}$, so that $\boldsymbol{P} = \boldsymbol{R}$. □

P 395, Fig. 4.10. On the second figure of the first line, identify the axes as ξ^1, ξ^2, on the second figure of the second line, identify the axes as η^1, η^2, and on the third figures of the first and second lines, identify the axes as y^1, y^2.

P 396, Ex. 5.8. Read"Find the principal stretches, the principal axes, the images of the principal axes, and the rotation tensor for ... ."

P 399, Exercise 5.20. Replace the first sentence with "Let (s, θ, ϕ) be spherical coordinates for $\mathbb{E}^3$, so that $\tilde{\boldsymbol{z}}(\mathbf{x}) = s[\sin\theta(\cos\phi\, \boldsymbol{i}_1 + \sin\phi\, \boldsymbol{i}_2) + \cos\theta\, \boldsymbol{i}_3]$."

P 399, Exercise 5.22. Replace the bold-face Roman symbols with bold-face italic symbols and replace ϵ with ε.

P 400. Delete the last term in (6.2). Replace (6.3) with

$$(6.3) \qquad \tfrac{d}{dt} m(\boldsymbol{p}(\boldsymbol{z}, t)) = 0.$$

(In Sec. 15, we learn how to get an explicit representation for this derivative under favorable assumptions.)

P 401, three lines after (6.5). Replace $\tilde{\rho}$ with $\breve{\rho}$.

P 403, L −9. After "These" insert "latter".

P 403. Shift the penultimate paragraph "In the rest ... " to P 402 right after Exercise 7.7.

P 403. Before the final paragraph insert the following material:

An important example of a surface traction on a part $\mathcal{S}$ of $\partial\mathcal{B}$ is that due a *hydrostatic pressure*, which is characterized by a constant intensity P of force per unit *actual* area of $\boldsymbol{p}(\mathcal{S}, t)$ acting in the direction opposite to the outer normal to $\boldsymbol{p}(\mathcal{S}, t)$. To find the corresponding $\boldsymbol{t}$, we let (s^1, s^2) be surface coordinates for $\mathcal{S}$ so that $\mathcal{S}$, assumed to be continuously differentiable, is described by $(s^1, s^2) \mapsto \tilde{\boldsymbol{z}}(s^1, s^2)$. Let $\tilde{\mathbf{s}}$ be the inverse of $\tilde{\boldsymbol{z}}$.

7.12B. Exercise. Let $\boldsymbol{\nu}(\boldsymbol{z})$ be the unit outer normal to $\partial\mathcal{B}$ at a material point $\boldsymbol{z}$ of $\mathcal{S}$, so that the differential surface area $da(\boldsymbol{z})$ of $\mathcal{S}$ at $\boldsymbol{z}$ satisfies

$$\boldsymbol{\nu}(\boldsymbol{z})\, da(\boldsymbol{z}) = \frac{\partial \tilde{\boldsymbol{z}}}{\partial s^1}(s^1, s^2) \times \frac{\partial \tilde{\boldsymbol{z}}}{\partial s^2}(s^1, s^2)\, ds^1\, ds^2$$

(provided that the coordinate curves are properly oriented). Show that the resultant force on $\mathcal{S}$ due to the hydrostatic pressure is

$$
\begin{aligned}
-P\int_{\tilde{\mathbf{s}}(\mathcal{S})}\left[\frac{\partial \boldsymbol{p}(\tilde{\boldsymbol{z}}(s^1,s^2),t)}{\partial \boldsymbol{z}}\cdot\frac{\partial \tilde{\boldsymbol{z}}}{\partial s^1}(s^1,s^2)\right]\times\left[\frac{\partial \boldsymbol{p}(\tilde{\boldsymbol{z}}(s^1,s^2),t)}{\partial \boldsymbol{z}}\cdot\frac{\partial \tilde{\boldsymbol{z}}}{\partial s^2}(s^1,s^2)\right]ds^1\,ds^2 \\
= -P\int_{\mathcal{S}}(\det \boldsymbol{F}(\boldsymbol{z},t))\boldsymbol{F}^{-*}(\boldsymbol{z},t)\cdot\boldsymbol{\nu}(\boldsymbol{z})\,da(\boldsymbol{z}) = \int_{\mathcal{S}}\boldsymbol{t}(\boldsymbol{z},t;\partial\mathcal{B})\,da(\boldsymbol{z}),
\end{aligned}
$$

so that $\boldsymbol{t}(\boldsymbol{z},t;\partial\mathcal{B}) = -P(\det \boldsymbol{F}(\boldsymbol{z},t))\boldsymbol{F}^{-*}(\boldsymbol{z},t)$ for hydrostatic pressure. (This formula can be readily adjusted to handle normal forces that vary with position, like those treated in Section III.5.)

P 404, Thm 7.14. Replace "If $\boldsymbol{t}(\cdot,t,\boldsymbol{\nu}(\boldsymbol{z}))\ldots$ on $\mathcal{B}$," with "If $\boldsymbol{t}(\cdot,t,\boldsymbol{\nu})\ldots$ on $\mathcal{B}$ for each $\boldsymbol{\nu}$,"

P 407, Fig. 7.32. Replace ξ_3 with $\tilde{\boldsymbol{\xi}}_3$.

P 408, three lines after (7.34). Add "Such a basis is seldom the most useful for $\boldsymbol{T}$."

P 409, L 5. Replace $\partial\mathcal{B}$ with $\mathcal{S}_3$.

P 409, (8.5). Delete the arguments of $\boldsymbol{p}$.

P 409, L –2. After "traction" add "at $\boldsymbol{z}$".

P 418, L –3. Replace "*differential*" with "*strain-rate*".

P 419. Before L 1 insert "(Some authors refer to these materials as being of *differential* type and others refer to them as being of *rate* type.) In L 5, replace "*differential*" with "*strain-rate*". In L10 replace "*rate*" with "*stress-rate*". After (10.8) insert "(Some authors refer to these materials as being of *rate* type.) After (10.10), replace "rate" with "stress-rate".

P 420, L 7. Replace "a change in length δ" with "a change δ in length".

P 420, L –11. Insert "This means that the responsibility to account for the physical effects of rigid motions devolves on the acceleration terms in the equations of motion. In particular, in the example just treated of the motion of a spring on a turntable, the rigid rotation produces the centrifugal acceleration $-r\dot{\phi}^2$."

P 423. Two lines after (12.20, replace "For Simplicity … assume that" with "As in Sec. 10, we take $\boldsymbol{Q} = \boldsymbol{R}^*$ to conclude that".

P 425, Ex. 12.14. Replace "Sidoroff (1978), and T. C. T. Ting (1985)." with "and T. C. T. Ting (1985, 1996)."

P 426, 427, (12.26a,b), (12.27). Replace $\tilde{\boldsymbol{C}}$ with $\check{\boldsymbol{C}}$.

P 428. In the middle of the paragraph following (12.33d), replace $p = -\frac{1}{3}\boldsymbol{S} : C$ with $p = -\frac{1}{3}\boldsymbol{S} : \boldsymbol{C}$.

P 429, in the line following (12.33b), after *pressure*, insert " The exact physical meaning of p inheres in this equation and, in particular, in the precise manner in which $\hat{\boldsymbol{S}}_{\mathrm{A}}$ is chosen."

P 429, five and six lines after (12.41b). Replace "*are rather*" with "*rather are*".

P 432, L 9. Delete space after the dash.

P 438. Replace the period at the end of equation (13.13) with a comma. At the beginning of the next line add "provided that $\boldsymbol{C}$ is invertible."

P 442, L 5. Delete "a million".

P 442–443. After the sentence containing (14.3) insert:

"We take the dot product of the equation of motion (7.21) with $\boldsymbol{p}_t$ to obtain

$$\frac{d}{dt}[\tfrac{1}{2}|\boldsymbol{p}_t|^2] = (\nabla \cdot \boldsymbol{T}^*) \cdot \boldsymbol{p}_t + \boldsymbol{f} \cdot \boldsymbol{p}_t = \nabla \cdot (\boldsymbol{T}^* \cdot \boldsymbol{p}_t) - \boldsymbol{T} : \boldsymbol{p}_{\boldsymbol{z}t} + \boldsymbol{f} \cdot \boldsymbol{p}_t, \tag{14.3$'$}$$

where the rightmost term is computed by introducing components. Integrating (14.3′) and using the Divergence Theorem, we obtain

$$\frac{d}{dt}K = P - \int_{\mathcal{B}} \boldsymbol{T} : \boldsymbol{F}_t \, dv, \tag{14.3$''$}$$

which says that $\int_{\mathcal{B}} \boldsymbol{T} : \boldsymbol{F}_t \, dv$ is the part of the power not converted into motion.

To get a coordinate-free proof of the second equality of (14.3′), we use (XI.2.19a) to obtain"

Now move to the position following this material the passage beginning with (4.10) and ending with the sentence following (4.14). In the line following (14.12), do not split $\boldsymbol{T} : \boldsymbol{F}_t$. Finally, continue with the paragraph following (14.3).

P 442, L –4. Replace the last three words with "motions, changes temperature, and causes".

P 443. After (14.6), insert "The energy balance says what happens to $\int_{\mathcal{B}} \boldsymbol{T} : \boldsymbol{F}_t \, dv$: It is either stored or converted to heat flow." Delete the two lines following (14.7). In the next line replace "We ... (7.21)" with "We substitute (14.3′) into (14.7)".

P 444, L7. Replace "We" with the following passage:

"When $Q < 0$, it follows that $P > \frac{d}{dt}K + \frac{d}{dt}E$, by the energy balance (4.6). Thus the excess of mechanical power over energy in the body produces heat (as in scenario (i)). There is no restriction on this rate. When $Q > 0$, heat is being added to the body, and $P < \frac{d}{dt}K + \frac{d}{dt}E$. In particular, when $P < 0$, power is extracted from the body. But there are limits to how much power the body can produce. (In scenario (ii), the interface produces no power, and it would produce no power even if it were heated a little.) In short, the energy balance describes the intraconvertibility of heat, power, and energy. But each body is limited in the rate at which it can convert heat into mechanical power. To capture such phenomena, we may"

P 444, (14.16). Replace Q with $Q[\mathcal{B}]$.

P 444, three lines after (14.16). Add "The temperature θ appearing in (14.16) should be interpreted as the temperature at which heat is supplied. Its role in (14.16) suggests that the bound at which heat can be supplied increases with temperature (in consonance with the range of temperatures suitable for heating a house). The presence of θ in (14.16) indicates that it is intimately related to the concept of entropy."

P 445. In the two lines following Exercise 14.26, delete "$, \boldsymbol{T}, \boldsymbol{q}, \eta, \psi$".

P 450, L 4. Replace "$\in \mathcal{B}$." with "$\in \mathcal{B}$, so that $\boldsymbol{y} = \boldsymbol{p}(\boldsymbol{z}, t)$ if and only $\boldsymbol{z} = \boldsymbol{q}(\boldsymbol{y}, t)$."

P 451, 15.8. Replace the first two lines of Lagrange's Criterion with "Let $\boldsymbol{p}$ be a motion of a body $\mathcal{B}$ and let $\mathcal{S}(t)$ be the configuration at time t in $\boldsymbol{p}(\mathcal{B}, t)$ of a moving surface. Let $\mathcal{S}(t)$ be defined by $\psi(\boldsymbol{y}, t) = 0$. Then $\mathcal{S}(t)$ is a material surface for all t if and only if".

Replace the first sentence of the proof with "If $\mathcal{S}(t)$ is a material surface for all t, then there is a function φ such that $\varphi(\boldsymbol{z}) = \psi(\boldsymbol{p}(\boldsymbol{z}, t), t)$ for all t. (This says that the material points lying on $\mathcal{S}(\tau)$ for some given τ do not vary with t.)"

P 455. At the end of the paragraph containing (15.34), read "$\boldsymbol{\Sigma} = (-p + \lambda \operatorname{tr} \boldsymbol{D})\boldsymbol{I} + 2\mu\boldsymbol{D} \ldots$ ".

Chapter XIII

P 460, Exercise 2.6. Replace $\boldsymbol{H} = \boldsymbol{Q} - \boldsymbol{I}$ with $\alpha\boldsymbol{H} = \boldsymbol{Q} - \boldsymbol{I}$.

P 463. Add the following material to Exercise 2.20:

Many computations for nonlinear problems are carried out for materials with constitutive functions of the form

$$\hat{\boldsymbol{S}}(\boldsymbol{C}, \boldsymbol{z}) = \tfrac{1}{2}\lambda(\boldsymbol{z}) \operatorname{tr}(\boldsymbol{C} - \boldsymbol{I})\boldsymbol{I} + \mu(\boldsymbol{z})(\boldsymbol{C} - \boldsymbol{I})$$

where the given Lamé coefficients λ and μ satisfy $\mu > 0$, $3\lambda + 2\mu > 0$ everywhere. (Cf. (XII.13.14), (15.14), and Ex. 15.15.) Prove that this constitutive function does *not* satisfy the Strong Ellipticity Condition.

P 471, Exercise 3.8. After "**Exercise.**" add "Let (3.1) hold."

P 471, Exercise 3.9. After "**Exercise.**" add "Let (3.1) hold."

P 477, Exercise 4.14. Delete "or cylindrical".

P 477. Replace Exercise 4.15 with

4.15. Exercise. Cavitation. For the full cylinder $\mathcal{B}$ given by (4.5a), let $\alpha = 1 = \delta$, $\beta = 0 = \gamma$, $g = 0 = h$. Show that if the material is hyperelastic, then the constitutive functions for $\overset{+}{T}{}_1{}^1$ and $\overset{+}{T}{}_2{}^2$ for this restricted class of deformations can be taken to have the form of restrictions of their representations from (3.20) with $\psi_1 = 0$ and with ψ_0 depending only on $\frac{f}{s} + \frac{s}{f}$. (i) Let $f(1)$ be a prescribed number greater than 1. Show that the equilibrium equations have a solution with $f(0) > 0$ and with the traction on the surface $s = 1$ finite provided that $\overset{+}{T}{}_2{}^2$ is integrable. Find simple conditions on the constitutive functions ψ_0 and ψ_1 that ensure this integrability. This solution describes a configuration in which the axis of the cylinder opens into a cylindrical cavity. (ii) Let $T_1{}^1$ be a prescribed positive number λ for $s = 1$. Show that for each λ the equilibrium equations admit the trivial solution given by $f(s) = s$ for all s. If there is a solution with a cavity, i.e., if $\overset{+}{T}{}_2{}^2$ is integrable when $f(0) > 0$, then it is reasonable to require the vanishing of the Cauchy traction on the boundary of the cavity. This condition, which relates the radius $f(0)$ of the cavity to λ, gives the branch of nontrivial solutions. Characterize the bifurcation point λ^0

at which cavitation is initiated. Compute it when $\psi_0(\mathrm{I}_{\boldsymbol{C}}) = \frac{sf}{s^2+f^2}$. (For an in-depth treatment of this problem, see Ball (1982) [[Enter in Refs]]. For a treatment of cavitation for compressible bodies, see the discussion in Sec. 7 and works cited there.)

P 480. In the title of Section 6, change "Cube" to "Block".

P 488. After Problem 6.43, add

6.44. Exercise. Formulate the equilibrium equations for deformations of the form (6.1) for incompressible bodies, and find the form of f in terms of $\alpha, \beta, \gamma, \delta$ and a constant of integration. Formulate and solve analogs of Exs. 4.12 and 4.16.

P 507, (9.7). Replace O with $\boldsymbol{O}$.

P 510. In (10.3) replace $w_{,1}^2 + w_{,2}^2$ with ${w_{,1}}^2 + {w_{,2}}^2$.

P 511, L 2. Insert the following paragraph:

To solve (10.4), (10.5) we may suppose that $p_{,3} = 0$. Then (10.5) reduces to a quasilinear partial differential equation for the scalar unknown w. Once this is solved, p is immediately found from (10.4). Note that for hyperelastic materials,

$$\overset{+}{T}{}_3{}^{\beta} = 2\left[\frac{\partial W^\dagger}{\partial \mathrm{I}_{\boldsymbol{C}}} + \frac{\partial W^\dagger}{\partial \mathrm{II}_{\boldsymbol{C}}}\right] w_{,\beta}$$

where the argument of $\overset{+}{T}{}_3{}^{\beta}$ is (10.2a), and the arguments of $W^\dagger$ are $\mathrm{I}_{\boldsymbol{C}}, \mathrm{II}_{\boldsymbol{C}}$ of (10.3). These are the stresses that appear in the fundamental partial differential equation (10.5). The form of $\overset{+}{T}{}_3{}^{\beta}$ for homogeneous isotropic materials is unrestricted:

Exercise. Prove that if

$$\gamma \mapsto H(\gamma^2) \equiv \frac{\partial W^\dagger}{\partial \mathrm{I}_{\boldsymbol{C}}}(3+\gamma^2, 3+\gamma^2) + \frac{\partial W^\dagger}{\partial \mathrm{II}_{\boldsymbol{C}}}(3+\gamma^2, 3+\gamma^2)$$

is given, then there always exists a $W^\dagger$ satisfying (10.11).

P 511, L 6. Add the sentence "The problem treated in Section II.7 is essentially an antiplane problem, and Theorem I.7.8 is analogous to Theorem 10.9."

P 529. After the first paragraph, add:

15.15. Exercise. Find necessary and sufficient conditions on λ and μ for the quadratic form $H^{ijkl}E^{(1)}_{ij}E^{(1)}_{kl}$ to be positive-definite when (15.14) holds.

Chapter XIV

P 532, (1.2). Read

$$\forall\, \mathbf{x} \in \mathcal{A} \equiv \tilde{\mathbf{x}}(\mathcal{B}).$$

P 534, (1.17). Replace $\tilde{\mathbf{x}}(\mathcal{B})$ with $\mathcal{A}$. Replace $\mathbf{q}(\tilde{z}(\mathbf{x}), t)$ with $\mathbf{q}(\mathbf{x}, t)$, $\bar{\boldsymbol{\tau}}(\tilde{z}(\mathbf{x})$, with $\bar{\boldsymbol{\tau}}(\mathbf{x},$, and $\boldsymbol{p}_1(\tilde{z}(\mathbf{x}))$ with $\boldsymbol{p}_1(\mathbf{x})$.

P 534, (1.18). Replace this equation with

$$\overset{\scriptscriptstyle\vartriangle}{\boldsymbol{p}}(\mathbf{x}, t) = \frac{\partial \bar{\boldsymbol{p}}}{\partial \mathsf{q}}(\mathbf{x}, t, \mathsf{q}(\mathbf{x}, t)) \cdot \mathsf{a}(\mathbf{x}, t) \quad \text{for}(\mathbf{x}, t) \in \partial \mathcal{A} \times [0, \infty), \tag{1.18}$$

P 534. Replace the five lines following (1.18) with

for all (nice enough) a. The function $\overset{\scriptscriptstyle\vartriangle}{\boldsymbol{p}}$ is a typical tangent vector to the constraint manifold defined by the boundary condition (1.15a). Note that the remaining boundary condition (1.15b) and initial condition (1.16b) are incorporated into (1.17).

P 535. Replace title of Sec. 2 with "Induced Rod Theories".

P 535. Replace (2.2) with

$$\mathcal{A}(s) \equiv \tilde{\mathbf{x}}(\mathcal{B}(s)) \equiv \{\mathbf{x} \in \mathcal{A} : \mathbf{x} = (x^1, x^2, s)\}. \tag{2.2}$$

P 535. In the line following (2.3) replace "three" with "two".

P 535 L –9. Replace "We assume" with "For unconstrained three-dimensional materials we assume".

P 535 L –3. Before "We require" insert, "Below, we shall show how to modify (2.5) to handle three-dimensional material constraints."

P 535 (2.7). Replace final comma with a period. After (2.7) add "$\boldsymbol{\pi}$ is allowed to depend on τ in order to accommodate such time-dependent boundary conditions on $\mathcal{L}$."

P 536. Delete the last sentence of the first paragraph.

P 536 (2.8). Replace $\mathcal{B}$ with $\mathcal{A}$.

P 537. In (2.17a) replace $\boldsymbol{\tau}^k$ with $\boldsymbol{\tau}^k_{\mathrm{L}}$. In (2.18) and (2.19) replace x with $\mathbf{x}$.

P 537. After the paragraph containing (2.15), insert the following material:

We postpone the treatment of material constraints such as incompressibility until Sec. 6B because it involves some unexpected subleties. Most of the resulting theories nevertheless have a structure like that discussed in this section.

P 539, L 14. Replace (1.19) with (1.20).

P 539, L –10. Add the comment:

P 539, Exercise 2.37. Replace N with I. At the beginning of L –4 insert "(so that $N = \frac{1}{2}(I+1)(I+2)$)". In (2.38) divide $(x^1)^\alpha (x^2)^\beta$ by $(\alpha + \beta)!$.

P 540. In the line following (2.39b) add "(cf. (XII.12.81))" after "distributions". Add "(see Antman (1966))" to the penultimate sentence of Section 2.

P 544. Replace the sentence containing (3.26) with

"If, however, we substitute (3.3c) into (3.15)–(3.24), we immediately obtain

$$\boldsymbol{m} = \boldsymbol{d}_\alpha \times \boldsymbol{m}^\alpha, \qquad \boldsymbol{l} = \boldsymbol{d}_\alpha \times \boldsymbol{f}^\alpha, \tag{3.26a,b}$$

$$\boldsymbol{b}^\alpha = \rho I^\alpha \boldsymbol{r}_t + \varepsilon^{\alpha\gamma}\varepsilon^{\beta\delta}\rho J_{\gamma\delta}\partial_t \boldsymbol{d}_\beta, \qquad \boldsymbol{c}^\alpha = \mathbf{0}, \tag{3.26c,d}$$

$$\boldsymbol{h} = \boldsymbol{d}_\alpha \times \varepsilon^{\alpha\gamma}\varepsilon^{\beta\delta}\rho J_{\gamma\delta}\partial_t \boldsymbol{d}_\beta \tag{3.26e}$$

where ρI^α and $\rho J_{\gamma\delta}$ are defined in (VIII.4.6), (VIII.4.7)."

Now move the last three lines of P 544 right after these equations and renumber the result. Next append the two sentences following (3.26), but with "Thus" replaced by "By using (3.29) we show that".

After this material, insert as a new paragraph the material in small type at the top of P 545, but with "(3.29)" inserted after the second word "consequence"

P 544. In (3.29), replace $\boldsymbol{r}'$ with $\boldsymbol{r}_s$.

P 545, L 9. Replace "treatment" with "treatments".

P 545. At the end of Sec. 3, insert

3.30. Exercise. Assume that the $\boldsymbol{d}_k$ are orthonormal. Use (3.3c) to construct from (1.17) the special Cosserat theory of Chap. VIII. (There is no apparent practical virtue in generalizing (3.3c).) Use the fact that $\overset{\vartriangle}{\boldsymbol{d}}_k$ has the form $\boldsymbol{d}_k \times \boldsymbol{\omega}$.

P 558. In the caption of Fig. 6.27, replace "corresponding to" with "indicates".

P 558. Replace (7.1) with

$$(s,t) \mapsto \boldsymbol{d}_0(s,t), \boldsymbol{d}_1(s,t), \ldots, \boldsymbol{d}_K(s,t), \quad \boldsymbol{d}_0(s,t) \equiv \boldsymbol{r}(s,t).$$

P 567, (9.18). Replace $da(\mathbf{s})$ with da and replace $\bar{\mathbf{b}}(\mathbf{s})\cdot\overset{\vartriangle}{\mathbf{u}}(\mathbf{s},0)\,da(\mathbf{s})$ with $\bar{\mathbf{b}}\cdot\overset{\vartriangle}{\mathbf{u}}(\cdot,0)\,da$.

P 573, L 1, 2. Move comma from beginning of second line.

P 574. Add

$$\begin{aligned} \Gamma\Delta \equiv \mathring{r} \int_{h_1}^{h_2} \left(\boldsymbol{T}_{\mathrm{A}} \cdot \mathring{\boldsymbol{a}}\right) \cdot \left(\frac{\partial \boldsymbol{\kappa}}{\partial \boldsymbol{d}} \cdot \boldsymbol{b}\right)_s (1 - \varphi\mathring{\sigma})\varphi_{,3}\, dx^3 \\ + \mathring{r}\boldsymbol{b} \cdot \int_{h_1}^{h_2} \left(\boldsymbol{T}_{\mathrm{A}} \cdot \mathring{\boldsymbol{b}}\right) \cdot \frac{\partial^2 \boldsymbol{\kappa}}{\partial \boldsymbol{d} \partial x^3} (1 - \varphi\mathring{\mu})(1 - \varphi\mathring{\sigma})\varphi_{,3}\, dx^3. \end{aligned} \tag{11.14f}$$

P 583, L 8. Replace "with a smooth boundary." with "having a piecewise C^1 boundary."

P 584, L 10. Add the following material:

"We start by characterizing $\boldsymbol{\gamma}$. Let $\lambda \mapsto \check{\mathbf{s}}(\lambda)$ be a sufficiently smooth (absolutely continuous) curve in int $\mathcal{M}$ where λ is the arc-length parameter. Its unit tangent vector is $\frac{d\check{\mathbf{s}}}{d\lambda} \equiv \left(\frac{d\check{s}^1}{d\lambda}, \frac{d\check{s}^2}{d\lambda}\right)$ and (one of) its unit normal vectors is $\boldsymbol{\beta} := \left(\frac{d\check{s}^2}{d\lambda}, -\frac{d\check{s}^1}{d\lambda}\right)$."

Insert the sentence following (13.13), and continue with

"Then the corresponding unit normal $\boldsymbol{\gamma}$ to the curve $\lambda \mapsto \mathring{\boldsymbol{r}}(\check{\mathbf{s}}(\lambda))$ is given by

$$
\begin{aligned}
\boldsymbol{\gamma}((\check{\mathbf{s}}(\lambda)) &= \frac{\frac{d}{d\lambda}\mathring{\boldsymbol{r}}(\check{\mathbf{s}}(\lambda)) \times \mathring{\boldsymbol{d}}(\check{\mathbf{s}}(\lambda))}{|\frac{d}{d\lambda}\mathring{\boldsymbol{r}}(\check{\mathbf{s}}(\lambda))|} \\
&= \frac{[\beta_2(\check{\mathbf{s}}(\lambda))\mathring{\boldsymbol{r}}_{,1}(\check{\mathbf{s}}(\lambda)) - \beta_1(\check{\mathbf{s}}(\lambda))\mathring{\boldsymbol{r}}_{,2}(\check{\mathbf{s}}(\lambda))] \times \mathring{\boldsymbol{d}}(\check{\mathbf{s}}(\lambda))}{|\beta_2(\check{\mathbf{s}}(\lambda))\mathring{\boldsymbol{r}}_{,1}(\check{\mathbf{s}}(\lambda)) - \beta_1(\check{\mathbf{s}}(\lambda))\mathring{\boldsymbol{r}}_{,2}(\check{\mathbf{s}}(\lambda))|} \\
&= \frac{\Gamma(\check{\mathbf{s}}(\lambda))\beta_\alpha(\check{\mathbf{s}}(\lambda))\bar{\boldsymbol{g}}^\alpha(\check{\mathbf{s}}(\lambda))}{|\beta_2(\check{\mathbf{s}}(\lambda))\mathring{\boldsymbol{r}}_{,1}(\check{\mathbf{s}}(\lambda)) - \beta_1(\check{\mathbf{s}}(\lambda))\mathring{\boldsymbol{r}}_{,2}(\check{\mathbf{s}}(\lambda))|} \\
&=: \gamma_\alpha(\check{\mathbf{s}}(\lambda))\bar{\boldsymbol{g}}^\alpha(\check{\mathbf{s}}(\lambda)) \quad \text{where} \quad \gamma_\alpha = \boldsymbol{\gamma} \cdot \mathring{\boldsymbol{r}}_{,\alpha}.
\end{aligned}
\tag{13.13}
$$

"

Now continue with the third paragraph on page 584 beginning with "Let us choose". Number the next display (13.14a,b,c). Here and below replace the parameter α by λ.

Replace the material from (13.13) to the end of Ex. 13.14 with

"given by (13.13) with $\check{\mathbf{s}}$ taken to (13.14b)."

P 584, L –5. Insert "reference" before "length". Number the last display (13.15).

P 585, (13.18). Insert commas before subscripts on $\mathring{\boldsymbol{r}}$ and replace $|\bar{\boldsymbol{g}}^1 \cdot \bar{\boldsymbol{g}}^1|$ and $|\bar{\boldsymbol{g}}^2 \cdot \bar{\boldsymbol{g}}^2|$ by $|\bar{\boldsymbol{g}}^1|$ and $|\bar{\boldsymbol{g}}^2|$. Near the end of the first line of (13.21), insert the missing parenthesis. Delete the last sentence on this page.

P 586. After (13.23) insert the folloing material:

"These equations agree with (10.4) and (10.19). In no place in our derivation of these equations did we exploit the fact that $|\boldsymbol{d}| = 1$. We now consider this case, in which there are five geometrical unknowns, namely, the components of $\boldsymbol{r}$ and two functions specifying the orientation of the unit vector $\boldsymbol{d}$. When the resultants in these equations are prescribed by constitutive equations, we ostensibly have six equations for these five unknowns. Our resolution of this dilemma differs from that of Section 10 because we do not have access to the machinery used there.

P 590, L 4. Move comma.

P 591. In the first line of the last paragraph of Sec. 13, delete "the second integral of".

Chapter XV

P 605, L 3. After "(2.2)" insert "Thus, no matter what the interpretation of $\boldsymbol{\Pi}$ is, no matter what its tensorial character, and no matter how it transforms under rigid motions, the system (2.3), (2.4) describes materials invariant under rigid motions."

P 606. In the second paragraph, add the argument t to $\boldsymbol{C}$ and $\boldsymbol{\Pi}$ where it is missing.

P 608. Replace the material from L 5 "Now we ... " to the end of the proof $\square$ with "Thus $\mathcal{A}$ annihilates every tensor that is orthogonal to $\gamma_{\boldsymbol{C}}$. To show that this implies that $\mathcal{A}$ has the form $\boldsymbol{M}\gamma_{\boldsymbol{C}}$, we observe that the analog of (2.14), (2.15) in which $\mathcal{A}$ is replaced with a second-order tensor, and $\boldsymbol{C}_t$ and $\gamma_{\boldsymbol{C}}$ are replaced with vectors $\boldsymbol{x}$ and $\boldsymbol{g} \neq \boldsymbol{0}$ is

$$(2.16) \qquad \boldsymbol{A} \cdot \boldsymbol{x} = \boldsymbol{0} \quad \forall \boldsymbol{x} \quad \text{such that} \quad \boldsymbol{x} \cdot \boldsymbol{g} = 0.$$

Now let $\boldsymbol{e}_3 \equiv \boldsymbol{g}/|\boldsymbol{g}|$ and choose $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ such that $\{\boldsymbol{e}_k\}$ is an orthonormal basis for $\mathbb{E}^3$. Then we can represent $\boldsymbol{A}$ as $\boldsymbol{A} = A_{kl}\boldsymbol{e}_k\boldsymbol{e}_l$. Condition (2.16) implies that $0 = \boldsymbol{A} \cdot \boldsymbol{e}_\alpha = A_{k\alpha}\boldsymbol{e}_k$, $\alpha = 1, 2$. The independence of $\{\boldsymbol{e}_k\}$ implies that $A_{k\alpha} = 0$, so that $\boldsymbol{A} = A_{k3}\boldsymbol{e}_k\boldsymbol{e}_3 = \boldsymbol{mg}$, $\boldsymbol{m} \equiv A_{k3}\boldsymbol{e}_k/|\boldsymbol{g}|$. To carry over this proof to (2.14), (2.15), we need only prove the following generalization of Proposition XI.1.16: *If* $\{\boldsymbol{A}_{ij}\}$ *and* $\{\boldsymbol{B}_{kl}\}$ *are bases for* Sym, *then* $\{\boldsymbol{A}_{ij}\boldsymbol{B}_{kl}\}$ *is a basis for* $\mathcal{L}$(Sym,Sym). (This proof could have been replaced with a suitable application of the general Multiplier Rule XVII.2.24.) $\square$"

P 611. Replace the last two lines with "Let

$$(3.9b) \qquad \mathsf{H} = \lambda\frac{\partial\gamma}{\partial\mathsf{U}} + \mathsf{J} \quad \text{where } \mathsf{J} \circ \frac{\partial\gamma}{\partial\mathsf{U}} = 0.$$

Substituting (3.9b) into (3.8) we obtain

$$(3.9c) \qquad \mathsf{H} = \lambda\frac{\partial\gamma}{\partial\mathsf{U}}, \quad \lambda \geq 0.$$

In summary, we obtain from (3.8) and (3.9) that"

P 612. Delete the first five lines. In (3.10b) replace $\hat{\boldsymbol{S}}$ with $\frac{1}{2}\hat{\boldsymbol{S}}$.

P 614, (4.11) and two lines thereafter. Replace γ with g.

P 618, L 8. Replace "The set of" with "For our deformations, the set of nontrivial".

P 618, three lines after (5.11). Before "We assume" insert "By substituting this expression into (5.10), we find that h is independent of x_1, x_2."

P 621, (5.27). Replace g_1 and g_2 with γ_1 and γ_2.

P 621. After the sentence containing (5.29), add "If $I_3 = 0$, we get a similar inconsistency."

P 625, the two lines following (6.8). Replace "*region ... plastically.*" with "*region* or that the system behaves *elastically.* (In this case there is no motion of the cylinder relative to the piston, so that all the deformation is due to that of the elastic spring.) Otherwise, we say that $x(t)$ is in the *plastic region* or that the system behaves *plastically.*" In the next paragraph, rephrase statements (i) and (ii) thus: "(i) if $\gamma(x(t), \pi(t)) < 0$, then $\dot{\pi}(t) = 0$, and (ii) if $\gamma(x(t), \pi(t)) = 0$, then $\dot{\pi}(t) \ldots \hat{N}(x(t) - \pi(t)) - \beta$."

P 625, L –3. After "here" insert "(since $\gamma \leq 0$)" and replace $|\dot{\pi}|(t) = 0$ with $|\dot{\pi}(t)| = 0$.

P 626, Figure 6.13. Replace the label X on the abscissa with x.

P 626. After the second sentence of the first paragraph insert "This figure corresponds to the experimental situation in which the mass point M of Figure 6.1 is replaced with a force F necessarily equal to $\hat{N}(x-\pi)$. The experiment might consist in prescribing $x(\cdot)$ and monitoring $F(\cdot)$. Although $\hat{N}$ is defined on a semi-infinite interval, the nature of our system implies that the accessible range of N is just $[-\alpha+\beta, \alpha+\beta]$, as shown. Suppose that x is increased to a value $\hat{N}^{-1}(\alpha+\beta)+\pi_0$, where $\pi_0 > 0$. Initially, as x increases to $N^{-1}(\alpha+\beta)$, $N = \hat{N}(x)$ because $\gamma(x,\pi) < 0$ so $\dot{\pi} = 0$, by (6.11). As x is increased from $N^{-1}(\alpha+\beta)$ to $N^{-1}(\alpha+\beta)+\pi_0$, N must stay at the value $\alpha+\beta$, i.e., γ must stay at 0, because if not, (6.8) and (6.11) would imply that γ would exceed 0. Now suppose that x is reduced from $N^{-1}(\alpha+\beta)+\pi_0$ to π_0. Then $N = \hat{N}(x-\pi_0)$, because if not, N would have to satisfy $N = \alpha+\beta$, so that (6.11) would contradict (6.22)."

P 626, (6.14). Replace $<$ with $\leq$ and replace $=$ with $>$.

P 626, four lines after (6.14). Replace the two sentences "When N ... by (6.14)." with "In this case, Fig. 6.13 does not hold. We do know that when $\dot{\pi} = 0$, N as a function of x is still given by a curve of the form $N = N(x-\pi)$, but with N likely to take values in an interval larger than $[-\alpha+\beta, \alpha+\beta]$."

P 626, L –17. Replace "dashpots in series" with "dashpots in parallel". Delete the last sentence "(For ... " of this paragraph and the first sentence "Alternatively, ... " of the next paragraph, and combine the two paragraphs. After this combined paragraph insert the following paragraph:

"Suppose we replace the device in Fig. 6.1 with one in which our two different kinds of dashpots operate in series. For example, we could have the spring and the viscous dashpot act in parallel, connecting a massless bar to the wall. This massless bar could then be connected to the mass point with a dashpot exerting Coulomb friction. Then viscosity acts all the time. (In the study of corresponding continua, as in Fig. 4.20, such dissipation plays an important role in the analysis of shocks.)"

Chapter XVI

P 642, (3.7b). Replace $\mu v_t(0,t)$ with $\mu v_t(1,t)$.

P 645. Above the last paragraph of Section 3, insert:

3.28. Exercise. Prove that the initial-boundary-value problem (3.1), (3.2), (3.7), (3.8) has at most one classical solution, by carrying out the following steps. (i) Let w_1 and w_2 be two such solutions, and let $\omega = w_1 - w_2$. Show that

$$\int_0^1 \rho A \omega_{tt}\omega_t \, ds + \mu\omega_{tt}(1,t)\omega_t(1,t) = -\int_0^1 \alpha\omega_s\omega_{st}\, ds - \int_0^1 \beta\omega_{st}^2\, ds \tag{3.29}$$

where

$$\begin{aligned} \alpha &\equiv \int_0^1 \hat{N}_y(\lambda\partial_s w_1 + (1-\lambda)\partial_s w_2, \lambda\partial_{st}w_1 + (1-\lambda)\partial_{st}w_2, s)\, d\lambda, \\ \beta &\equiv \int_0^1 \hat{N}_z(\lambda\partial_s w_1 + (1-\lambda)\partial_s w_2, \lambda\partial_{st}w_1 + (1-\lambda)\partial_{st}w_2, s)\, d\lambda. \end{aligned} \tag{3.30}$$

(ii) Use (3.9) to show that there is a continuous function Γ_1 such that

$$\begin{aligned}(3.31)\quad & \tfrac{1}{2}\int_0^1 \rho A\omega_t^2\,ds + \tfrac{1}{2}\mu\omega_t(1,t)^2 + m\int_0^t\int_0^1 \omega_{st}^2\,ds\,d\tau \\ & \leq \Gamma_1(t)\int_0^t\int_0^1 |\omega_s|\,|\omega_{st}|\,ds\,d\tau \leq \Gamma_1(t)\sqrt{\int_0^t\int_0^1 \omega_s^2\,ds\,d\tau}\sqrt{\int_0^t\int_0^1 \omega_{st}^2\,ds\,d\tau},\end{aligned}$$

whence there are continuous functions Γ_2 and Γ_3 such that

$$(3.32)\qquad \int_0^t\int_0^1 \omega_{st}^2\,ds\,d\tau \leq \Gamma_2(t)\int_0^t\int_0^1 \omega_s^2\,ds\,d\tau \leq \Gamma_3(t)\int_0^t\int_0^\tau\int_0^1 |\omega_s|^2\,ds\,d\eta\,d\tau.$$

(iii) Use the Gronwall inequality to complete the proof.

P 645. In the second sentence of the last paragraph of Section 3, replace "They" with "Antman & Seidman".

P 647, second equation of (4.11). Replace $\tilde{u}_\xi$ with $\tilde{v}_\xi$.

P 648, second equation of (4.12). Replace $\tilde{u}_\xi$ with $\tilde{v}_\xi$.

Chapter XVII

P 667. In paragraph containing (1.10) replace $\mathit{\Omega}$ with Ω.

P 668, one line before (2.3). After "*null space*" insert "(or *kernel*)".

P 670, Theorem 2.23 (iii). Replace $\mathcal{R}(\mathsf{A}^*)$ with $\mathcal{R}(\mathsf{A})$.

Chapter XIX

P 683, six lines after (1.1). After "there is" insert "a special way to assign the numbers ±1, leading to".

References

P 702. Replace the reference S. S. Antman & M. Lanza de Cristoforis (1995) with S. S. Antman & M. Lanza de Cristoforis (1997), Peculiar instabilities due to the clamping of shearable rods, *Int. J. Nonlin. Mech.* **32**, 31–54. (IV.1)

P 702. Replace the reference S. S. Antman & T. I. Seidman (1995) with S. S. Antman & T. I. Seidman (1996), Quasilinear hyperbolic-parabolic equations of nonlinear viscoelasticity, *J. Diff. Eqs.* **124**, 132–185.

P 709. Replace "P. J. Davis" with "P. J. Davies" and make the corresponding change on P 511.

P 709. Replace "E. E. Rossinger" with "E. E. Rosinger" and make the corresponding change on P 28.

P 722. Replace "to appear" in Müller & Spector with *Arch. Rational Mech. Anal.*, **131**, 1–66. In the next reference, replace "Yau (1995)" with "Yan (1994)" and replace "to appear" with "*Anal. Non Linéaire* **11**, 217–243.

P 729. Delete the reference to Sidoroff.

P 732. Insert

T. C. T. Ting (1996), New Expressions for the solution of the matrix equation $\mathbf{A}^T\mathbf{X} + \mathbf{X}\mathbf{A} = \mathbf{H}$, *J. Elasticity* **45**, 61–72. (XI.2, XII.12)

L. Trabucho & J. M. Viaño (1996), Mathematical Modelling of Rods, in *Handbook of Numerical Analysis*, Vol. IV, P. G. Ciarlet & J.-L. Lions, eds., North-Holland. (XIV.15)

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P 748 Insert "sine-Gordon equation", VIII.16".

P 749 Replace "Tensors" with "Tensor".

Acknowledgment

I am grateful to the following persons for corrections and helpful comments: J. M. Ball, S. Eberfeld, T. Frohman, J. Horváth, O. Lakkis, K. A. Rogers, Jinqiu Shao, J. G. Simmonds, Xiaobo Tan, R. Tucker, Roh Suan Tung, J. P. Wilber, L. von Wolfersdorf, S.-C. Yip.