Homework 10 – due 04/16/08

Math 601

43. Prove the following fact we used in lecture: suppose $E \supset F$ is an extension contained in anlgebraic closure $\overline{F}$ of F. Then the following are equivalent:

(1) $\sigma(E) = E$ for every $\sigma \in \mathrm{Aut}(\overline{F}/F)$ (i.e. E/F is *normal*).

(2) For every irreducible polynomial $f \in F[X]$, if it has one root in E, all its roots are in E.

44. Prove that a finite extension E/F is normal if and only if E is the splitting field of some $f \in F[X]$.

45. Dummit-Foote, 14.2, #17, 19.

46. Dummit-Foote, 14.2, #18, 21

47. Dummit-Foote, 14.4, #3.

48. Dummit-Foote, 14.6, #13.