Homework 13 – due 05/07/08

Math 601

59. (5 points) Let $\phi : V \to V$ and $\psi : W \to W$ be F-linear endomorphisms of finite-dimensional F-vector spaces V and W. Show that $\mathrm{Tr}(\phi \otimes \psi) = \mathrm{Tr}(\phi) \cdot \mathrm{Tr}(\psi)$.

60. Let F be a field. We say $u \in \mathrm{GL}_n(F)$ is *unipotent* if $u - 1$ is nilpotent, that is, $(u-1)^N = 0$ for some $N \geq 1$.

(a) Show that if $\mathrm{char}(F) = 0$, then the only finite-order unipotent element is the identity matrix.

(b) Use (a) to prove that if ρ is a representation of a finite group G over $\mathbb{C}$, then $\rho(g)$ is similar to a diagonal matrix. HINT: use the Jordan-normal form of $\rho(g)$.

(c) Show that the conclusion of (a) is false if $\mathrm{char}(F) = p$.

61. Dummit-Foote, 18.1, #16. (5 points)

62. Dummit-Foote, 18.3, #6-8 (30 points).