Homework 3 – due 02/20/08

Math 601

8. (10 points) Consider a commutative diagram

$$\begin{array}{ccccccccc}
0 & \longrightarrow & A_1^\bullet & \longrightarrow & B_1^\bullet & \longrightarrow & C_1^\bullet & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & A_2^\bullet & \longrightarrow & B_2^\bullet & \longrightarrow & C_2^\bullet & \longrightarrow & 0
\end{array}$$

where each row is an exact sequence of complexes, and each square commutes. Prove the *naturality of the boundary maps*, i.e., show that for each i the following commutes

$$\begin{array}{ccc}
\mathrm{H}^i(C_1^\bullet) & \xrightarrow{\delta_1} & \mathrm{H}^{i+1}(A_1^\bullet) \\
\downarrow & & \downarrow \\
\mathrm{H}^i(C_2^\bullet) & \xrightarrow{\delta_2} & \mathrm{H}^{i+1}(A_2^\bullet).
\end{array}$$

9. (5 points) Let $f : A \to B$ denote a morphism in an abelian category. In class we showed that the map

$$\ker(f) \to A$$

is a monomorphism. Prove in a similar way that the canonical map

$$B \to \mathrm{cok}(f)$$

is an epimorphism.

10. Dummit-Foote, 10.5, #2.

11. Dummit-Foote, 10.5, #6.

12. (15 points) Dummit-Foote, 10.5, #15.

13. Dummit-Foote, 10.5, #16.